Introduction to Reliability Engineering

	Month	Day	Time	Title
➡	February	9	Noon-2PM	Class Introduction-Overview, continuing into Prob-part 1
	February	23	Noon-2PM	Probability (Hint: Reliability is Applied Probability)-Part 1
	March	8	Noon-2PM	Probability (Hint: Reliability is Applied Probability)-Part 2
	March	22	Noon-2PM	Discrete and Continuous distributions-Part 1
	April	12	Noon-2PM	Discrete and Continuous distributions-Part 2
	April	26	Noon-2PM	Weibull Distribution -Part 1
	May	10	Noon-2PM	Weibull Distribution -Part 2
	May	24		
	June	14	Noon-2PM	Reliability Modeling -- parallel, series, redundant, standby systems
	June	28	Noon-2PM	FMEA-Part 1
	July	12	Noon-2PM	FMEA Part 2
	July	26	Noon-2PM	Reliability Testing -Part 1
	August	9	Noon-2PM	Reliability Testing -Part 2
	August	23	Noon-2PM	Reliability Testing -Part 3
	September	13	Noon-2PM	Reliability Allocations and Predictions, Reliability Growth-Part 1
	September	27	Noon-2PM	Reliability Allocations and Predictions, Reliability Growth-Part 2
	October	11	Noon-2PM	System Safety Analysis and link to Reliability-Part 1
	October	25	Noon-2PM	System Safety Analysis and link to Reliability-Part 2
	November	8	Noon-2PM	Maintainability & Human Reliability-Part 1
	November	22	Noon-2PM	Maintainability & Human Reliability-Part 2
	December	13	Noon-2PM	Other Topics (TBD)

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Reliability Overview

“Getting Ready!”

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Introduction to Reliability Engineering

What is Reliability?

- ***What do you think reliability is?***
- ***Textbook Definition:***

Reliability is the probability that an item will perform its intended function for a specified interval under stated conditions following prescribed procedures.

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Introduction to Reliability Engineering

What does that really mean?

- You must know the customer’s need.
- You must know the application and usage.
- You must know the expected usage life.
- You must know how to maintain reliability.

It means, ***“Quality over time.”***

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Introduction to Reliability Engineering

What does reliability look like?

This light bulb at a Fire Station in Livermore, California has been working since 1901! Livermore's Centennial Light Bulb

(over 1,077,000 hours)

url: http://www.centennialbulb.org/

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Introduction to Reliability Engineering

Let's recap….What is Reliability?

Classical Definition:

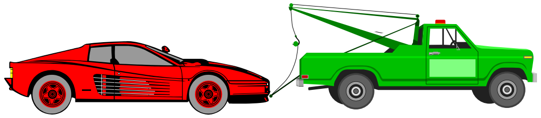
The probability that a system will perform its function adequately for a specified time when operating under stated conditions.

- Mission Reliability

  The probability the mission will be accomplished.

- Basic Reliability

  How often things break.



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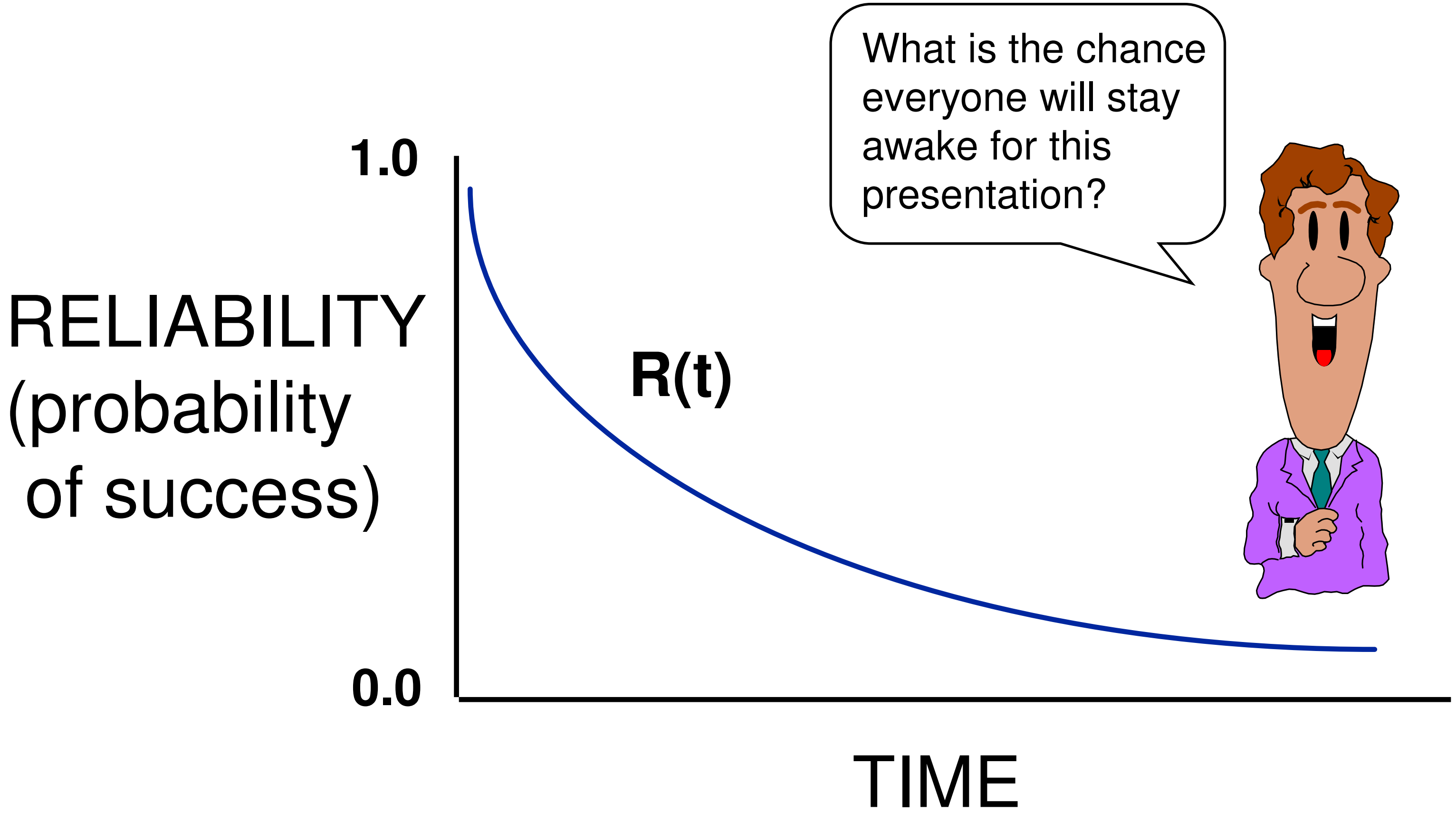
System/Mission Reliability is Probability Oriented



R(t)= reliability function

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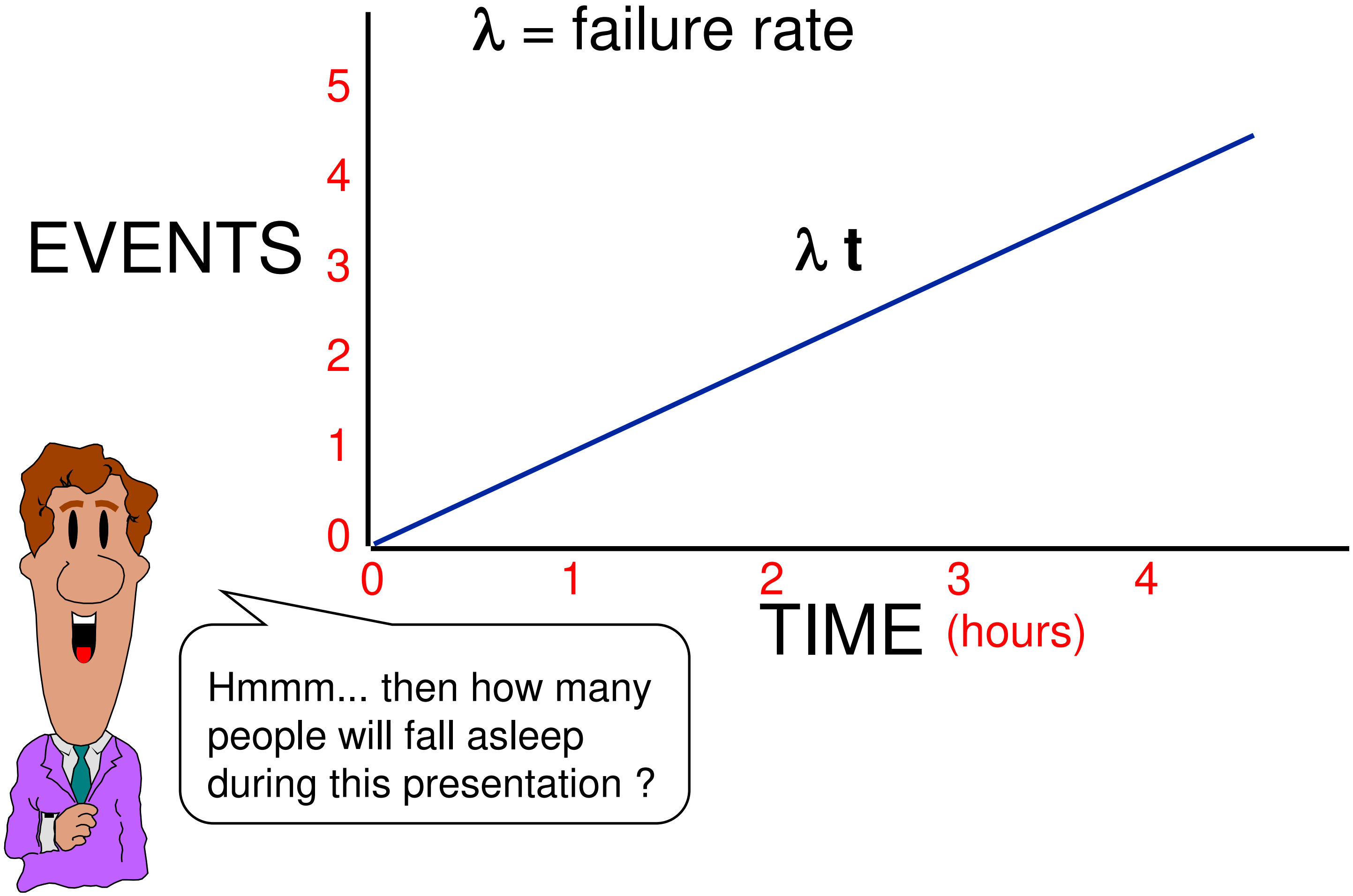
Basic Reliability is Event Oriented



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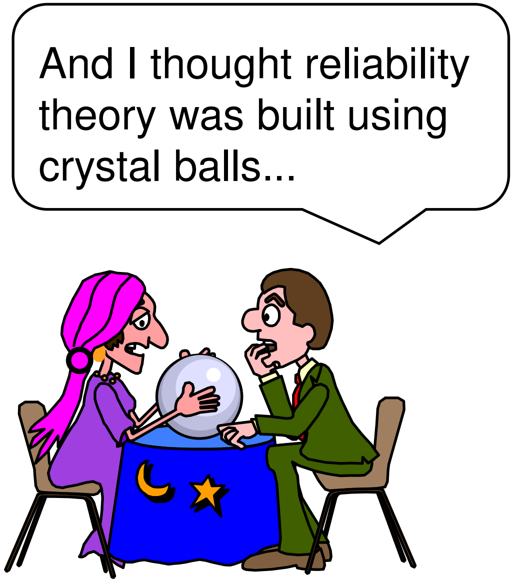
Introduction to Reliability Engineering

Reliability Theory is Built using Failure Rates

$$\text{Failure Rate} = \lambda = \frac{\text{Failures}}{\text{Time}}$$

$$\text{Mean Time Between Failure (MTBF)} = \frac{\text{Time}}{\text{Failures}}$$

$$\text{MTBF} = \frac{1}{\lambda}$$



EXAMPLE:

A unit operates for 100 hours during which it experiences 5 failures.

λ = 5 failures / 100 hours = 0.05 failures / hour

MTBF = 1 / λ = 20 hours

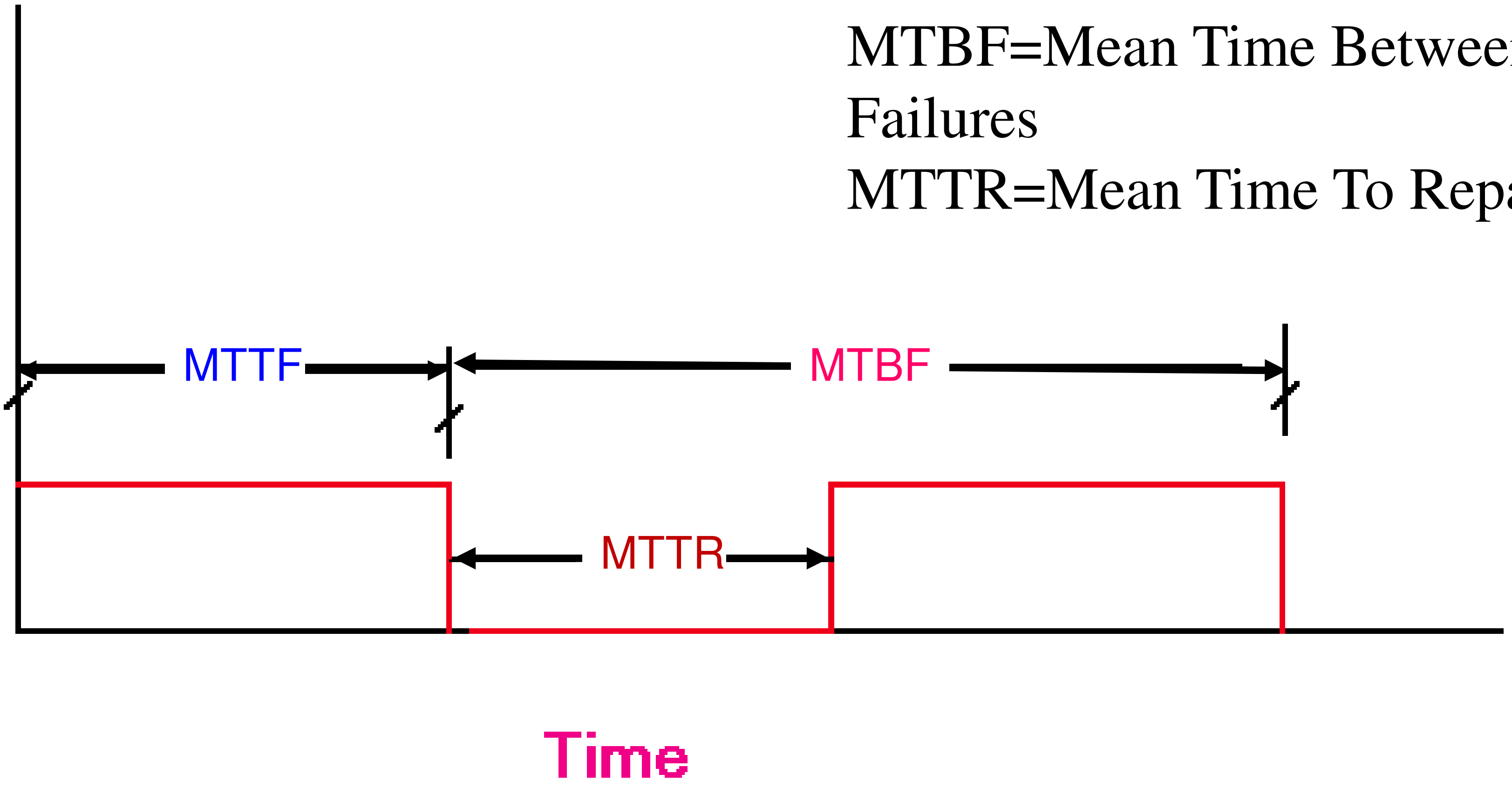
NOTE: The expected lifetime of a ***non-maintained*** component or its mean time to failure =MTTF (Mean Time To Failure).

Relationship Between MTTF, MTBF, MTTR

MTTF=Mean Time to Failure

MTBF=Mean Time Between Failures

MTTR=Mean Time To Repair



IN-CLASS Problem: Open EXCEL, Load "Part1 data&solutions.xls"

A System has a Mean Time Between Failure (MTBF) of 1500 hours that is relatively constant over time. The overhaul interval for the System is 4000 Cycles. The Cycle to hour ratio is 2:1. What is the reliability of the System to operate without failure to the overhaul?

Microsoft Excel Hints

1. *EXP(n) =* e^n
2. *LN(n) =* $\log_e$ *= natural logarithm*
3. $\therefore$ *LN(EXP(n)) = n*

If 100 Systems operate for the 4000 Cycle overhaul interval (Systems are repaired and returned to service if required), how many failures would be expected?

What would the MTBF of the System need to be to have a 50% chance of no failure to overhaul?

#1,#2,#3

RELIABILITY AS A PROBABILITY

If

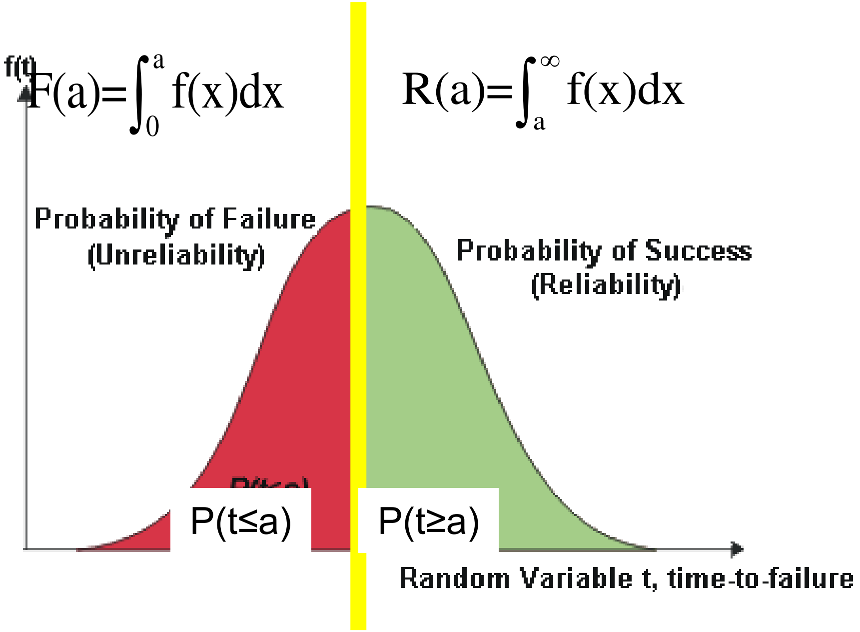
Probability(failure time $\geq$ a)=R(a),

and

Probability(failure time $\leq$ a)=F(a)

then,

$$R(a)=1-F(a)=1-\int_0^a f(x)dx=\int_a^\infty f(x)dx,$$



now, if the time to failure is constant, i.e. is described by an exponential funtion:

$$R(t) = 1 - F(t) = 1 - \int_0^t f(x)\,dx = \int_t^\infty f(x)\,dx = \int_t^\infty \lambda e^{-\lambda x} dx = e^{-\lambda t}$$

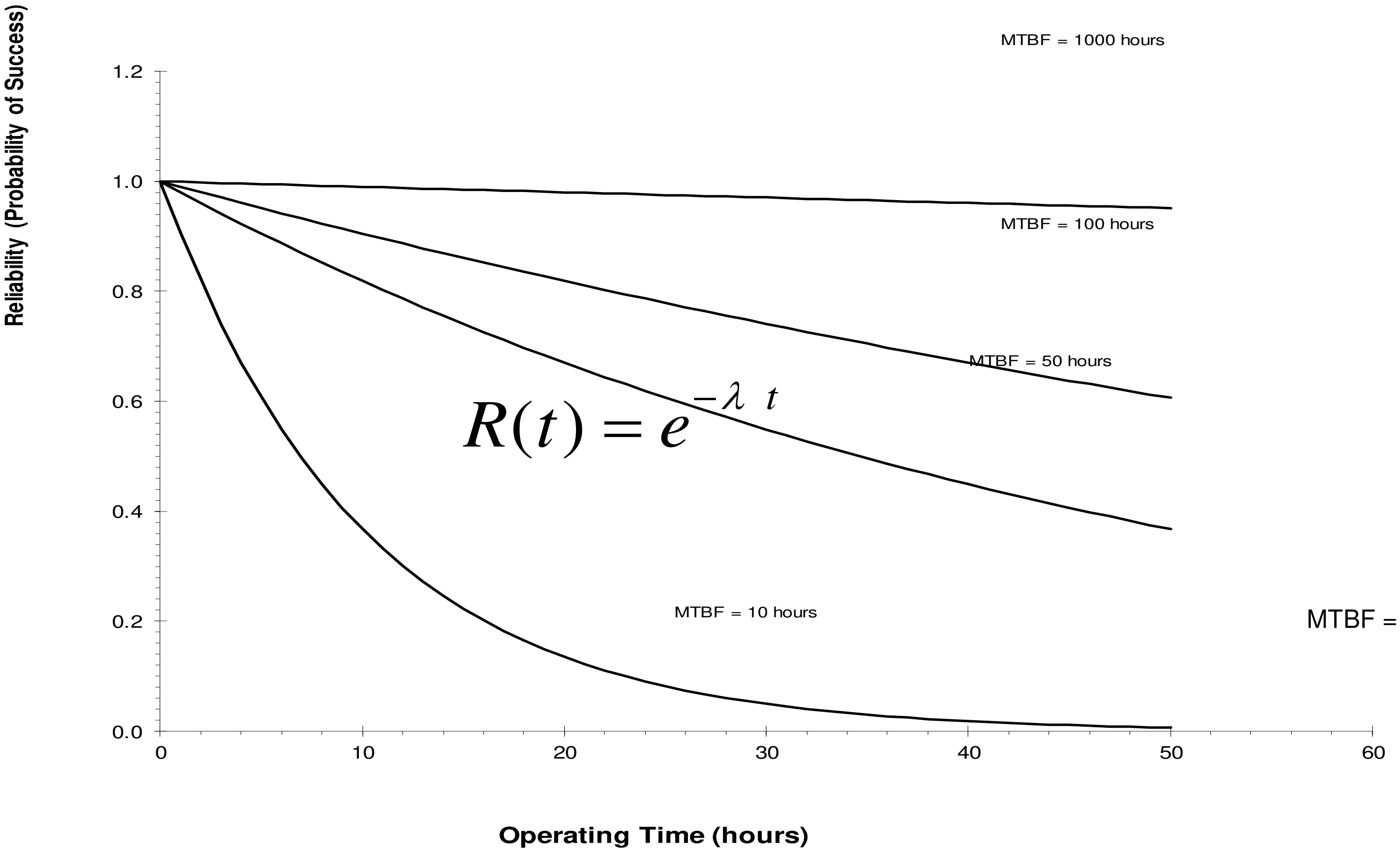
and,

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} = \lambda e^{-\lambda t}, t \geq 0$$

(we will cover other distributions later)

*Where f(x) is the pdf of the failure distribution(in this case, the exponential)

Success Probability Calculated using R(t)



$$R(t) = e^{-\lambda\ t}$$

$$\text{MTBF} = \frac{1}{\lambda}$$

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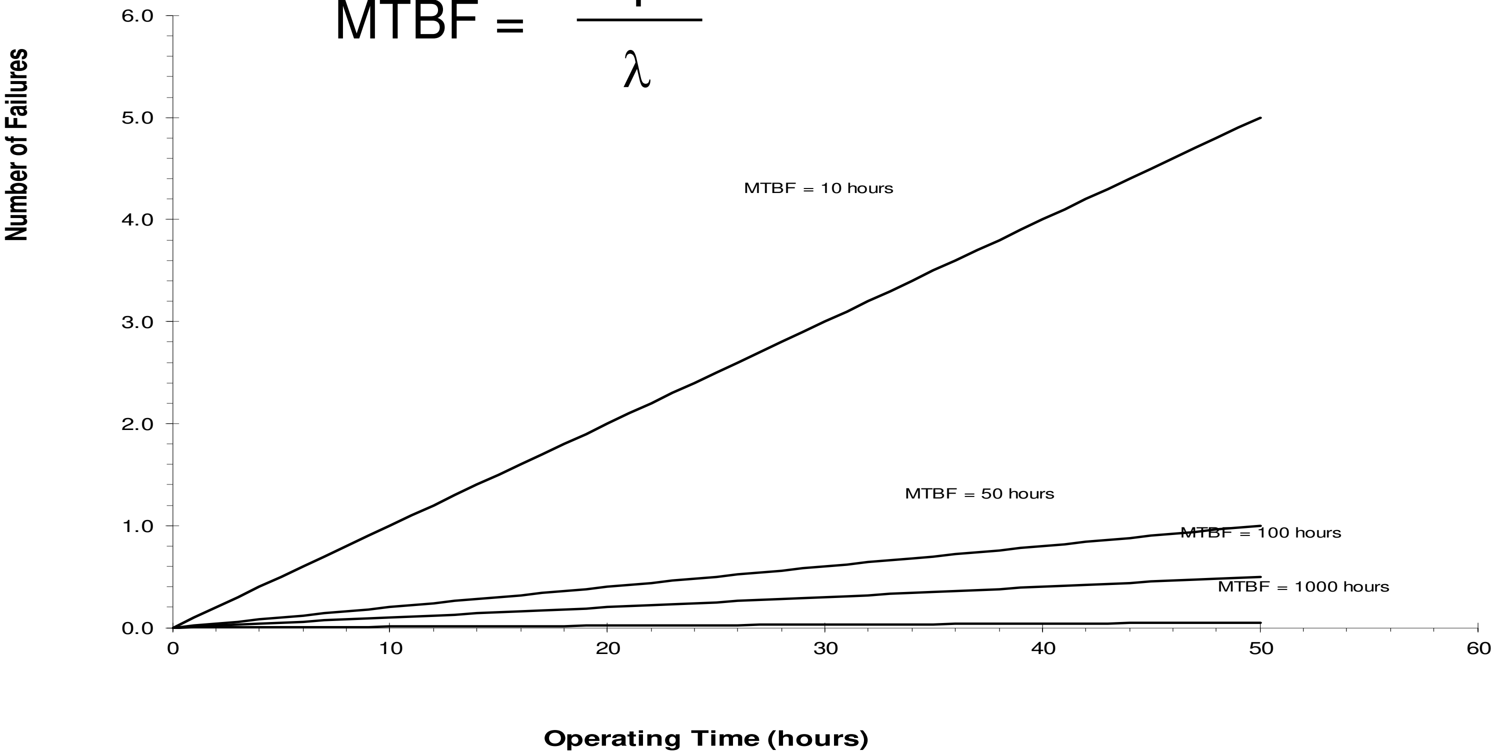
Number of Events Calculated using λ



$$\mathrm{MTBF} = \frac{1}{\lambda}$$

Introduction to Reliability Engineering

Probability

Reliability can be thought of as
"Applied Probability"

Tyche

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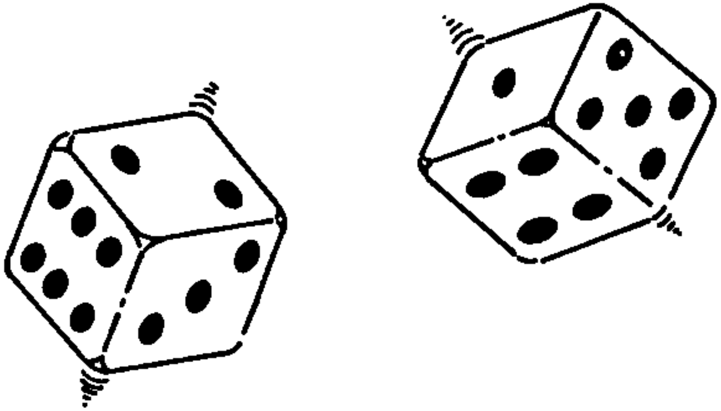
Introduction to Reliability Engineering

Outline of this section



- Introduction
- Probability of an event
- Laws of Probability
- Conditional probability & independence
- Permutations & Combinations
- Expected Value

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Introduction to Reliability Engineering

Introduction…thinking about uncertainty

Ordinary arithmetic and algebra and most of mathematics has exact answers (A+B=B+A, etc.)

Usually, in all the basic phenomena of life, biology, chemistry, physics (and hence engineering), the questions that really interest us are not clear-cut and do not have an exact answer.

e.g. Should I carry an umbrella today?

What are the Patriots chances of winning a superbowl?

Will this part make it to the next inspection?

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Introduction to Reliability Engineering

Introduction…thinking about uncertainty

Classical logic dealt with truth and falsehood (0 and 1). Nothing in between.

The logic of probability is not restricted to two truth values (0 and 1), it makes use of an infinite set of truth values, expressed in numbers lying between 0 and 1.

So, Probability theory is capable of considering situations in which we don't have enough information to permit the application of classical logic.

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Introduction to Reliability Engineering

Introduction

One major goal of statistical analysis:

Make inferences about the target population based on a sample. How?

=> Using PROBABILITY

Probability - provides a means of <u>quantifying uncertainty</u>.

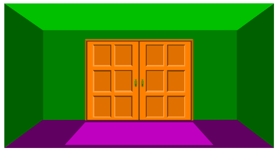
e.g. There is a 20% chance of rain today.

Most likely that the Chiefs will win the Superbowl.

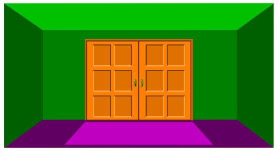
What is the chance of having a failed part?

Monty Hall's Let's Make A Deal

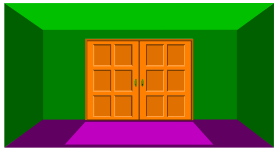
You're given the choice of three doors, behind one of which is a large amount of money. You pick one of the doors, but instead of telling you if you've won, you are shown $0 behind one of the remaining two doors. Then you are asked if you would like to switch doors to the other remaining door.... what are the probabilities involved, and what should you do?



Door No. 1



Door No. 2



Door No. 3

Outcomes	0	0	$
	0	$	0
	$	0	0

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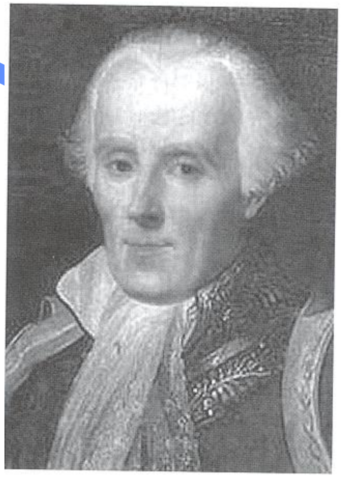
Introduction to Reliability Engineering

Introduction… The birth of Lady Luck

In 1654, the Chevalier de Mere, a gambler and philosopher in France, asked his friend Blaise Pascal why it was that betting on 1 “6” in 4 throws of a single die was better odds than 1 “double 6” in 24 throws of a pair of die?

Pascal(1623-1662)



Antoine Gombaud
chevalier de Méré
1607 - 1685

Fermat(1601-1665)

Introduction to Reliability Engineering

Introduction… The birth of Lady Luck

Pascal solved this problem, and while doing so he corresponded with Pierre de Fermat concerning his solutions.

This exchange of letters gave birth to probability theory.

…. We'll come back to this problem later.

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Introduction to Reliability Engineering

Introduction

* Coming up with models that are “Real World.”

* Models:

1. Equiprobable (tossing die, flipping coin)
2. Not equally probable (probability of rain, probability of failure, thumb tack drop)

This is one of the reasons you will see drawing colored balls out of urns used so often.

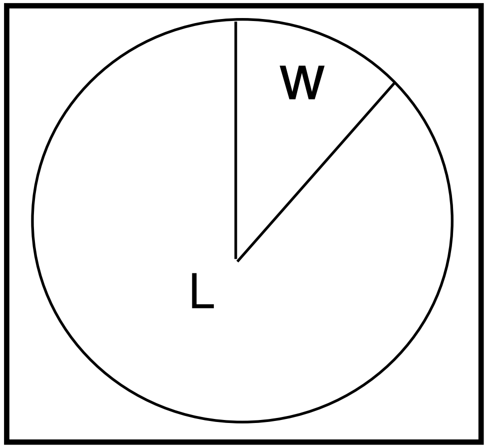
* Probability models of the real world can be very useful, but only as useful as the thought that went into them, probability models ARE NOT MAGIC.

Introduction to Reliability Engineering

Probability of an Event- Basic definitions

- Sample space (S) = set of all possible outcomes

e.g.



Spin once

Set of all possible outcomes=Sample Space(S)={W,L}

e.g. Toss a fair coin (assume 2 faces)

S={Heads, Tails}

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Sample Space… Examples

- Select 10 parts for inspection.

  Is each part o.k., or defective?

- Make parts with a target diameter.

  Diameter within spec, or out of spec?

- Launch an Atlas rocket with 2 RL10 engines in upper stage.

  Did the RL10's ignite?

Probability of an Event- Basic Definitions

- Outcome= an element (e) of the sample space.

e.g. In the Spin Once example:

W is an outcome,

L is also an outcome.

In the Toss coin example:

Tails is an outcome,

Heads is another outcome.

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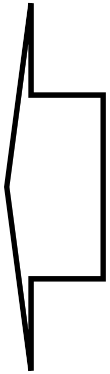
Outcomes… Examples

- Select 10 Parts for inspection.

Part	Defective	Good
1		X
2	X	
3		X
4		X
5		X
6		X
7	X	
8		X
9		X
10		X



10 outcomes

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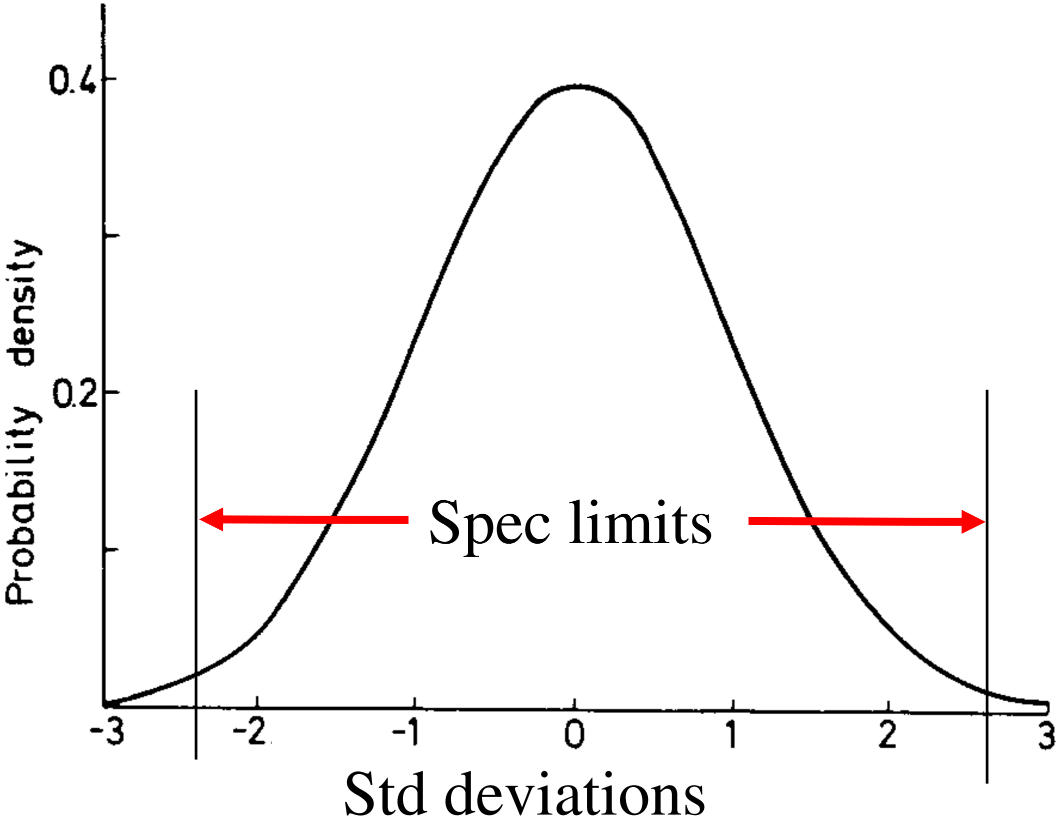
Outcomes… Examples(continued)

- Make parts with a target diameter



Diameter within spec….. or out of spec?

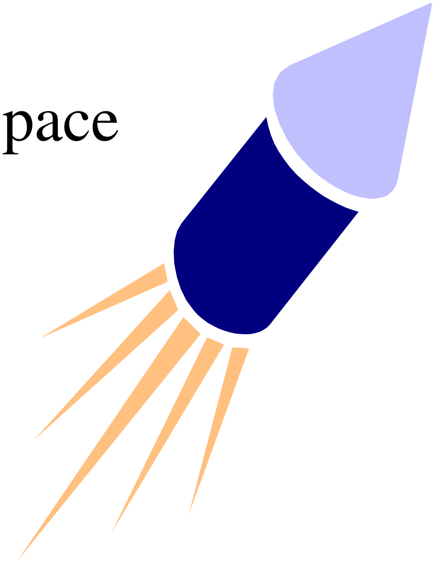
Introduction to Reliability Engineering

Outcomes… Examples(continued)

- Launch an Atlas rocket with 2 RL10 engines in upper stage.
  Did the RL10's ignite?

Suppose RL10 history is 290 in-space firings, no failures to ignite.

So, the probability that the RL10's will NOT ignite = 0??!!



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Probability of an Event- Basic Definitions

- Event= A subset of outcomes

e.g. Spin twice:

Sample space (S)={WW,WL,LW,LL}

Outcomes= WW,WL,LW,LL

Event = subset of outcomes

= symbol: A,B,C,D

For example,

Event A= at least one W= {WW,WL,LW}

Event B= both spins are L = {LL}

Probability of an Event- Basic Definitions

- **Probability of an Event**= a numerical value that represents the proportion of times the event is expected to occur when the experiment is repeated under identical conditions. Denoted **P(A).**

e.g. Spin Twice

Sample space(S)={WW,WL,LW,LL}

Event A=at least one W={WW,WL,LW}

If all outcomes are equally likely, => P(A) = 3/4

Event B= both spins are L={LL}

P(B)=?

So, if all outcomes are equally likely, then

$$\mathbf{P(B)} = \frac{\text{No. of outcomes in event B}}{\text{No. of total outcomes in Sample Space}} = 1/4$$

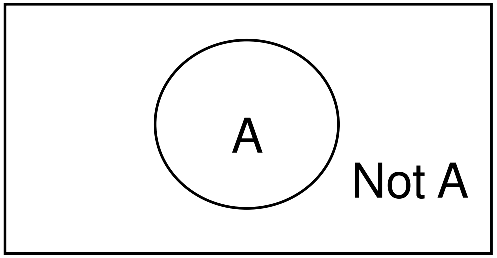
Probability of an Event- example

- Suppose that among 50 students in a class, 42 are right-handed and 8 are left-handed. If one student is randomly selected from the class, what is the probability that the selected student is left-handed? right-handed?

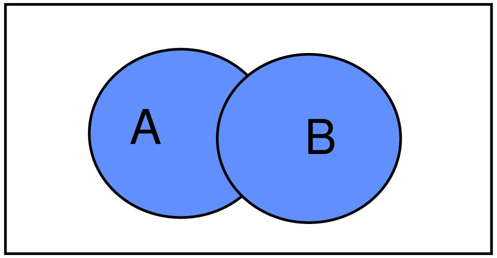
Laws of Probability

- Basic event operations:

$$(1)\ Complement\ of\ A(\overline{A}) = NOT\ in\ A$$



$$(2) Union\ A \cup B = A\ or\ B\ or\ Both$$



$A \cup B$ is the shaded area

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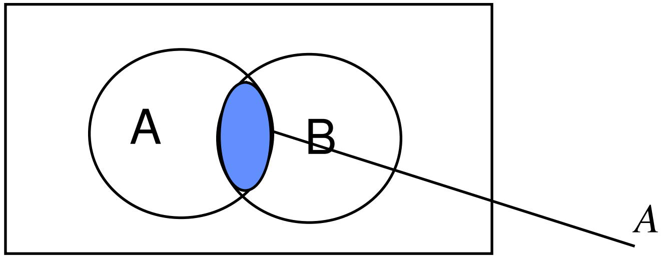
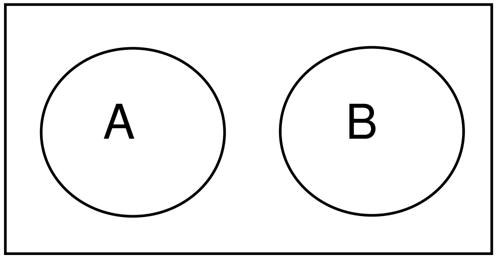
Laws of Probability

$$(3)\ \mathit{Intersection}\ \ A \cap B = A\ \ \mathit{and}\ \ B$$



$A \cap B$ *is this area*

$$(4)\ A\ \mathit{and}\ B\ \mathit{are\ Mutually\ Exclusive} = A\ \mathit{and}\ B\ \mathit{cannot\ occur\ simultaneously}$$



$A \cap B = \varnothing$

Introduction to Reliability Engineering

Laws of Probability.. example

- Spin Twice:

  Let A= at least one W={WW,WL,LW}

  Let B= both spins are L={LL}

=>

$$\overline{A} = \{LL\}$$

$$A \bigcup B = \{WW, WL, LW, LL\}$$

$$A \bigcap B = \varnothing \ \ (i.e., A, B \ are \ Mutually \ Exclusive)$$

Laws of Probability... in class exercise

Toss a fair coin twice,

Let A=tails at the second toss

Let B=at least one head

Give the compositions of the events:

	Probability
$S = \{(H,H),(H,T),(T,H),(T,T)\}$	=1.0
$A = \{(H,T),(T,T)\}$	=0.5
$B = \{(H,H),(H,T),(T,H)\}$	=0.75
$\overline{A} = \{(H,H),(T,H)\}$	=0.5
$A \bigcup B = S = \{(H,H),(H,T),(T,H),(T,T)\}$	=1.0
$A \bigcap B = \{(H,T)\}$	=0.25

$Are\ A\ and\ B\ Mutually\ Exclusive?$ NO

Laws of Probability

$$(5)\ \Pr obability(outcome) \geq 0$$

$$(6) \sum \Pr ob(all\ \ outcomes) = 1$$

$$So,\ \ all\ \ probabilities\ are\ between\ \ 0\ \ and\ \ 1.$$

$$(7) Law\ of\ \ complement:\ \ P(A) = 1 - P(\overline{A})$$

$$(8) Addition\ Law:\ \ P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$$

$$(9) If\ \ A, B\ are\ Mutually\ exclusive, P(A \bigcap B) = \varnothing$$

$$NOTE: If\ A\, and\ B\, are\, mutually\, exclusive\, P(A \bigcup B) = P(A) + P(B)$$

$$(10)\ If\ \ A, B\ are\ independent,\ \ then\ P(A \bigcap B) = P(A)\, P(B)$$

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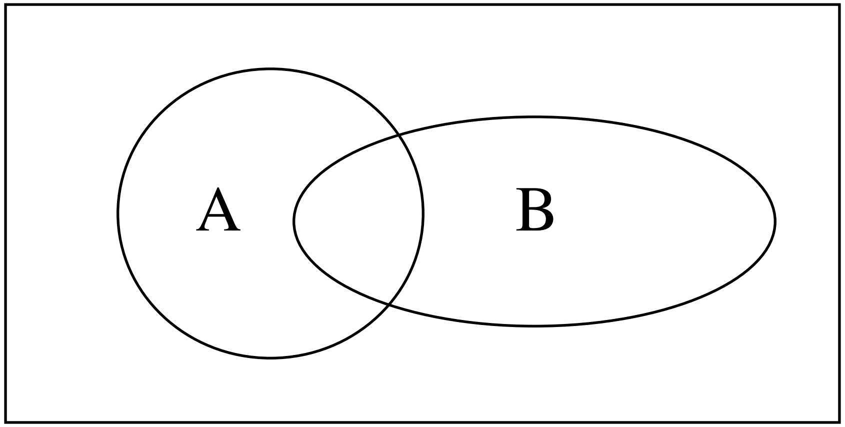
Example: DeMorgan's Rule

- Let A and B be two events in the same sample space.
- Consider two combinations of A and B:
    - The complement of the union of A and B
    - The intersection of the complements of A and B
- Do these two have the same probability?



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Introduction to Reliability Engineering

Laws of Probability...example

Toss coin twice (from last example)

P(A)=0.5

P(B)=0.75

By using the laws of probability,

=> $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B) = 0.5 + 0.75 - 0.25 = 1.0$

$$P(\overline{A}) = 1 - P(A) = 1 - 0.50 = 0.50$$

Remember:

Toss a fair coin twice,

Let A=tails at the second toss

Let B=at least one head

The compositions of the events:

$S = \{(H,H),(H,T),(T,H),(T,T)\}$

$\bar{A} = \{(H,T),(T,T)\}$

$B = \{(H,H),(H,T),(T,H)\}$

$A = \{(H,H),(T,H)\}$

$A \cup B = S$

$A \cap B = \{(H,T)\}$

A and B are NOT Mutually Exclusive!

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Independent events…an example

- Suppose we know that the probability that the width of a machine-made part will be within specified bounds is 0.90, and the probability that its length will be within the bounds is 0.95. Suppose further that 80% of the parts are within specified bounds for length *and* width. Are the two events "width within bounds" and "length within bounds" independent?

- We need to check whether $P(A \cap B) = P(A)P(B)$ in this case. Here $(0.90)(0.95)=0.855 \neq 0.80$. Therefore, the two events are not independent.

“Fun" Probability Problem #1

- An experiment results in one of the following events: E1, E2, E3, E4 or E5.
  - a. find P(E3) if P(E1)=.1, P(E2)=.3, P(E4)=.2, P(E5)=.1
  - b. find P(E3) if P(E1)=P(E3), P(E2)=.1, P(E4)=.2, P(E5)=.1
  - c. find P(E3) if P(E1)=P(E2)=P(E4)=P(E5)=.1

a. P(E3)=1-P(E1)-P(E2)-P(E4)-P(E5)=0.3

b. P(E3)=1-P(E3)-P(E2)-P(E4)-P(E5)=1-P(E3)-0.4=0.6-P(E3) hence, 2*P(E3)=0.6, and P(E3)=0.3

c. P(E3)= 1-P(E1)-P(E2)-P(E4)-P(E5)=0.6

"Fun" Probability Problem #2

- A nickel, dime and quarter are tossed, and the "up" faces are noted after each toss.
  a. List the events in the sample space for this experiment.
  b. Assign reasonable probabilities to the simple events.
  c. Find the probability of each of the following events.
    A: { at least one head appears}
    B: { Exactly one head appears}
    C: { The first toss is a head}

a.

Nickel	**Dime**	**Quarter**
N	D	Q
N	D	q
N	d	Q
n	D	Q
n	d	Q
n	D	q
N	d	q
n	d	q

b. For each coin toss=1/8

Probability
0.125
0.125
0.125
0.125
0.125
0.125
0.125
0.125
Sum= 1

c.

Event

A	B	C
Yes	No	Yes
Yes	No	Yes
Yes	No	Yes
Yes	No	No
Yes	Yes	No
Yes	Yes	No
Yes	Yes	Yes
No	No	No
P(A)=0.875	P(B)=0.375	P(C)=0.50

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“Fun” Probability Problem #3

- Consider the experiment composed of one roll of a fair die followed by one toss of a fair coin. List the events. Assign a probability to each event. Determine the probability of observing each of the following events:

  A: { 6 on the die; H on the coin}

  B: { Even number on the die; T on the coin}

  C: { Even number on the die}

  D: { T on the coin}

Sample space:						
Die	**Coin**	**Probability**	**A**	**B**	**C**	**D**
1	**H**	**0.083333333**	**No**	**No**	**No**	**No**
2	**H**	**0.083333333**	**No**	**No**	**Yes**	**No**
3	**H**	**0.083333333**	**No**	**No**	**No**	**No**
4	**H**	**0.083333333**	**No**	**No**	**Yes**	**No**
5	**H**	**0.083333333**	**No**	**No**	**No**	**No**
6	**H**	**0.083333333**	**Yes**	**No**	**Yes**	**No**
1	**T**	**0.083333333**	**No**	**No**	**No**	**Yes**
2	**T**	**0.083333333**	**No**	**Yes**	**Yes**	**Yes**
3	**T**	**0.083333333**	**No**	**No**	**No**	**Yes**
4	**T**	**0.083333333**	**No**	**Yes**	**Yes**	**Yes**
5	**T**	**0.083333333**	**No**	**No**	**No**	**Yes**
6	**T**	**0.083333333**	**No**	**Yes**	**Yes**	**Yes**
	Sum=	**1**	**P(A)=0.083333**	**P(B)=0.25**	**P(C)=0.50**	**P(D)=0.50**

“Fun” Probability Problem #4

- Two marbles are randomly drawn from a box containing two blue marbles and three red marbles. Determine the probability of observing each of the following events:

A: { Two blue marbles are drawn}
B: { A red and a blue marble are drawn}
C: { Two red marbles are drawn}

P(A)=P(blue on 1^{st})*P(blue on 2^{nd})= (2/5)*(1/4)=2/20

P(B)=P(blue on 1^{st})*P(red on 2^{nd}) + P(red on 1^{st})*P(blue on 2^{nd})=
(2/5)*(3/4)+(3/5)*(2/4)=(6/20)+6/20)=12/20

P(C)=P(red on 1^{st})*P(red on 2^{nd})=(3/5)*(2/4)=6/20

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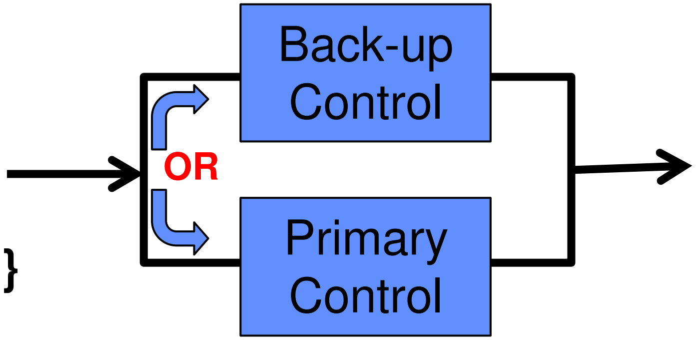
Introduction to Reliability Engineering

“Fun” Probability Problem #5

Brand X aircraft manufacturing company has a back-up control system that operates independently of the primary control. Each of the systems has a probability of 0.02 of failing on a particular mission. List the four events if we define the experiment to be observing the success or failure of the two operating systems. Now find the probability of the following events:

A: {Both systems function properly}
B: {At least one of the systems fail}
C: {Exactly one of the systems fails}
D: {At least one system functions properly}



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“Fun” Probability Problem #5(Solution)

Primary Control	Back-up control	P(System Working
P(Works)=.98	P(Works)=.98	(.98)*(.98)=.9604
P(Works)=.98	P(Does not work)=.02	(.98)*(.02)=.0196
P(Does not work)=.02	P(Works)=.98	(.02)*(.98)=.0196
P(Does not work)=.02	P(Does not work)=.02	(.02)*(.02)=.0004

So,

A: {Both systems function properly}= .9604

B: {At least one of the systems fail}=1-P(None fail)=1- .9604=.0396

C: {Exactly one of the systems fails}= .0196+.0196=.0392

D: {At least one system functions properly} =1- P(None work)=1- .0004=
.9996

Introduction to Reliability Engineering

"Fun" Probability Problem #6

- A string of lights for a Christmas tree has 12 bulbs on it. If any one of the lights doesn't work, the whole string goes out. If each of the lights has a probability of .95 of working when plugged in, what's the probability that the string lights? Assume all the lights work independently.
- Now assume each of the lights has a probability of .99 of working.

P(whole string lights)=P(1st lights)*P(2nd lights)*…*P(12th lights)=$(0.95)^{12}$~0.54

For P(light works=0.99):
P(whole string lights)=P(1st lights)*P(2nd lights)*…*P(12th lights)=$(0.99)^{12}$~0.886

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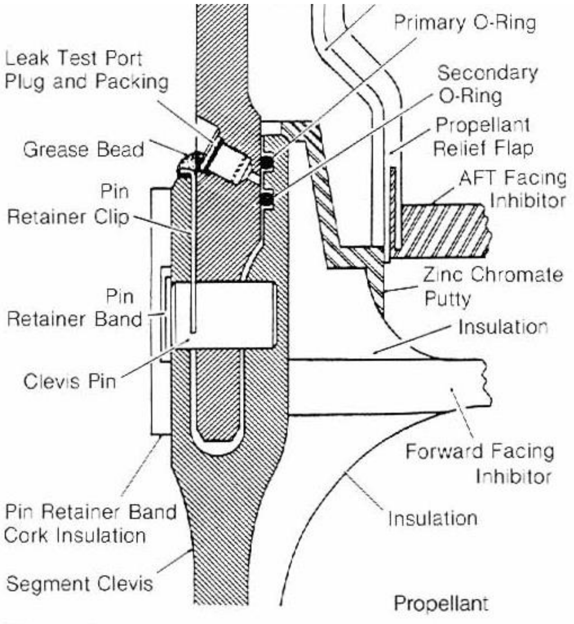
[redacted] Probability Problem #7

- The Space shuttle had 12 O-rings*. If any of the 12 rings fail, the shuttle would explode. If the probability of a single O-ring failing is .01, what's the probability of an explosion? Assume the rings operate independently.
- Recalculate if the probability of a single O-ring failing is .001.



If P(Oring failure)=.01 Prob at least 1 O-ring fails=1- Prob No O-rings fail
$=1-[P(O_1 \text{ doesn't fail}) * P(O_2 \text{ doesn't fail}) * \ldots * P(O_{12} \text{doesn't fail})]$
$=1-(.99)^{12} \approx .113$

If P(Oring failure)=.001 Prob at least 1 O-ring fails=1- Prob No O-rings fail
$=1-[P(O_1 \text{ doesn't fail}) * P(O_2 \text{ doesn't fail}) * \ldots * P(O_{12} \text{doesn't fail})]$
$=1-(.999)^{12} \approx .012$

* Each shuttle has two solid rocket boosters, each solid rocket booster has 4 "segments", hence 3 joints each with 2 rubber O-rings for a total of 12 O-rings.

Introduction to Reliability Engineering

Challenger (STS-51L) Disaster- Jan 28, 1986

In the back row from left to right: Ellison S. Onizuka, Sharon Christa McAuliffe, Greg Jarvis, and Judy Resnik. In the front row from left to right: Michael J. Smith, Dick Scobee, and Ron McNair.

STS-51L was the 25TH Shuttle firing:

If P(O ring failure)=1/25=0.04 Prob at least 1 O-ring fails=1- Prob No O-rings fail

$=1- [P(O_1 \text{ doesn't fail})^* P(O_2 \text{ doesn't fail})^* \ldots ^* P(O_{12} \text{doesn't fail})]$

$=1-(0.96)^{12} \approx .0.387$

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Introduction to Reliability Engineering

“Fun” Probability Problem #8

1. If P(A) is 0.5, P(B) is 0.2, and $P(A \cap B)$ is 0.1, what can be said about the relationship between the events A and B?
2. If $P(A \cap B)=0$, what can be said about the relationship between the events A and B?
3. If P(A)= 0.6, P(B)=0.3, and $P(A \cap B)=0.2$, are A and B independent?

Answers:

1. Events A & B are independent, since $P(A)*P(B)= P(A \cap B)$
2. Events A and B have nothing in common
3. No, since $P(A)*P(B) \neq P(A \cap B)$

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"Fun" Probability Problem #9

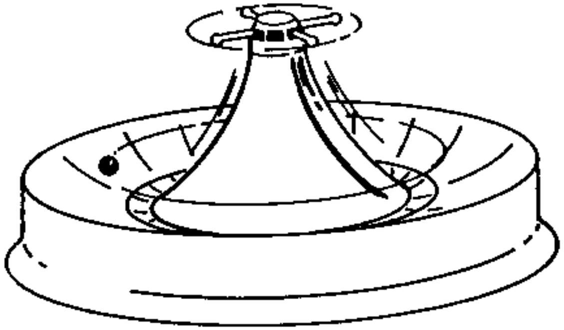
- Two warning systems are installed to monitor the operation of an engine part. Each will show a malfunction with probability 0.95. The probability that both warning systems work when a malfunction occurs is 0.99.

1. Are the two warning systems independent?
2. What's the probability that there will be no warning when the part malfunctions?

1. No, since P(1st shows malfunction)*P(2nd shows malfunction)=0.9025 ≠P(both show a malfunction)=0.99
2. P(No warning)=P(1st shows no warning)*P(2nd shows no warning)=0.05*0.05=0.0025

Introduction to Reliability Engineering

Roulette



- Assume a balanced or “fair” wheel;
  that is, each of the 38 possible outcomes
  1,2,3,...,36,0,00 is equally likely to occur.
- Now if the player places, say, $1 on number 5 and the steel ball comes to rest in slot number 5, he receives $36 from the house (his own $1 plus $35 winnings). If the ball comes to rest in any slot other than slot number 5, the house takes his $1.
- Answer the following:
  1. What’s the player’s chance of winning on a single bet?
  2. What’s the house’s chance of winning in a single bet?
  3. If the player repeatedly bets according to any scheme he chooses, what % of the total amount that he bets will he lose (or win) on the average?

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Roulette

1. Since all 38 possible outcomes 1,2,3,...,36,0,00 are equally likely on a fair wheel, the chance is 1 in 38 (1/38) that the player wins.
2. Since the house wins if any other outcome occurs, the house has 37 chances out of 38 (37/38) of winning.
3. Suppose the player places a $1 bet 380 times in succession; ignoring the sequence of numbers he picks, he wins 1/38 of the bets, or 10 bets. He loses 37/38 of the bets, or 370 bets. Each of the 10 wins gives him $35, for a total of $350; while his 370 losses lose him $370.

   Therefore, on the average, his net loss in 380 games is $20. So, in the long run, he is losing $20 out of every $380 bet or 5.3%.

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Laws of Probability

(11) Conditional probability, written P(A|B), means the probability that A occurs, given the "condition" that event B has already occurred.

$$P(A|B) = P(A \cap B) \,/\, P(B)$$

$$or$$

$$P(A \cap B) = P(B)P(A|B)$$

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Laws of Probability...conditional probability example

Toss a fair coin twice,

P(Tails at the second toss|at least one head)=?

Let A=Tails at the second toss

B=at least one head

$$P(A|B)=P(A\cap B)/P(B)= \frac{1/4}{3/4} = 1/3$$

S={(HH),(HT),(TH),(TT)}

So, P(A)=(HT),(TT)=1/2

P(B)=(HH),(HT),(TH)=3/4

P(A∩B)=1/4

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Introduction to Reliability Engineering

“Fun” Probability Problem #10

- 21% of the managers in a large engineering firm are at the (TSL)top salary level. It is further known that 40% of all managers at the firm are (W)women. Also, 6.4% of all managers are women *and* are at the top salary level, Recently, a question arose among executives at the firm as to whether there is any evidence of salary inequity.
- Do the percentages reported above provide any evidence of salary inequity?

$$P(TSL) = 0.21$$

$$P(W \cap TSL) = 0.064$$

$$P(W \mid TSL) = \frac{P(W \cap TSL)}{P(TSL)} = \frac{0.064}{0.21} \approx 0.3 \neq 0.4$$

OOPS, salary inequity

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Introduction to Reliability Engineering

Probability Trees

- A useful way of tackling many probability problems is to draw a *probability tree*.

- For example,

  A bag contains 8 white counters and 3 black counters. Two counters are drawn, one after the other. Find the probability of drawing one white and one black counter, in any order, (a) if the first counter is replaced, (b) if the first counter is not replaced.

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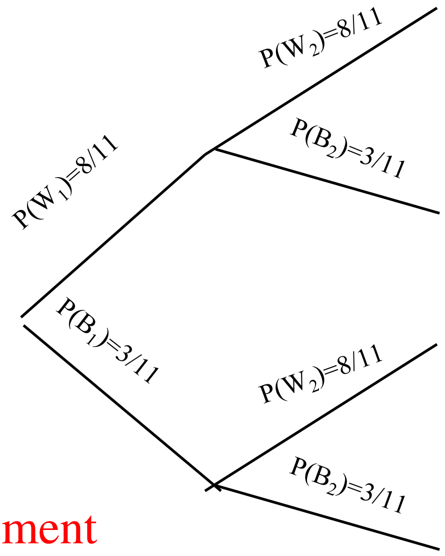
Probability Tree Example

- Let W_1 be the event ‘ a white counter is drawn first.’
- Let W_2 be the event ‘ a white counter is drawn second.’
- Let B_1 be the event ‘ a black counter is drawn first.’
- Let B_2 be the event ‘ a black counter is drawn second.’



$P(W_1 \cap W_2)=(8/11)(8/11)=64/121$

$P(W_1 \cap B_2)=(8/11)(3/11)=24/121$

$P(B_1 \cap W_2)=(3/11)(8/11)=24/121$

$P(B_1 \cap B_2)=(3/11)(3/11)=9/121$

With replacement

$$P(\text{drawing 1 white and 1 black counter})= P(W_1 \cap B_2)+ P(B_1 \cap W_2)$$
$$= 24/121 + 24/121 = 48/121$$

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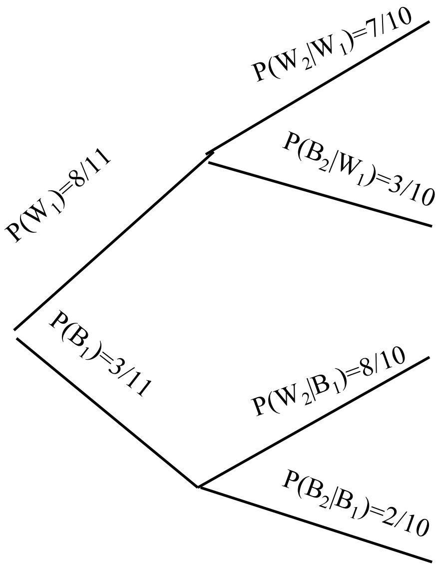
Probability Tree Example (continued)



$P(W_1 \cap W_2)=(8/11)(7/10)=56/110$

$P(W_1 \cap B_2)=(8/11)(3/10)=24/110$

$P(B_1 \cap W_2)=(3/11)(8/10)=24/110$

$P(B_1 \cap B_2)=(3/11)(2/10)=6/110$

Without replacement

P(drawing 1 white and 1 black counter)= $P(W_1 \cap B_2)+ P(B_1 \cap W_2)$
$= 24/110 + 24/110 = 24/55$

"Fun" Probability Problem #11

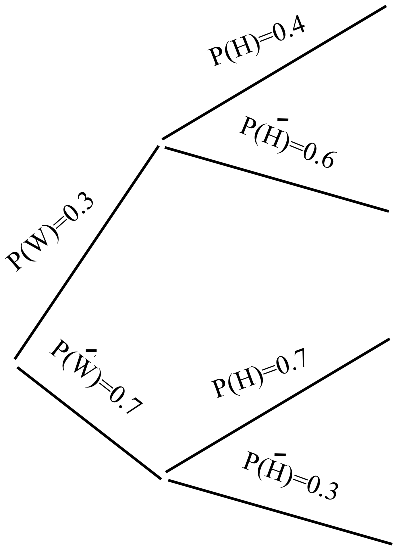
- The probability that a golfer hits the ball onto the green if it is windy as he strikes the ball is 0.4, and the corresponding probability if it is not windy as he strikes the ball is 0.7. The probability that it will be windy is 0.3.

  Find the probability that (a) he hits the ball on to the green, (b) it was not windy, given that he does not hit the ball onto the green.

Introduction to Reliability Engineering

Fun Probability Problem #11…solution

- Let W be the event ‘it is windy’, then $P(W)=0.3$ and $P(\overline{W})=1-0.3=0.7$
- Let H be the event ‘he hits the ball onto the green’ then $P(H|W)=0.4$ and $P(H|\overline{W})=0.7$



$P(W\cap H)=(0.3)(0.4)=0.12$

$P(W\cap \overline{H})=(0.3)(0.6)=0.18$

$P(\overline{W}\cap H)=(0.7)(0.7)=0.49$

$P(\overline{W}\cap \overline{H})=(0.7)(0.3)=0.21$

Therefore, the P(he hits the ball onto the green = 0.12+0.49=0.61

$$P(W|H)= \frac{P(W \cap H)}{P(H)}$$

$P(\overline{H})=1-P(H)=1-0.61=0.39$, so $P(\overline{W}|\overline{H})= 0.21/0.39 = \sim 0.54$
so, the prob it is not windy, given he does not hit ball on green,= 0.54

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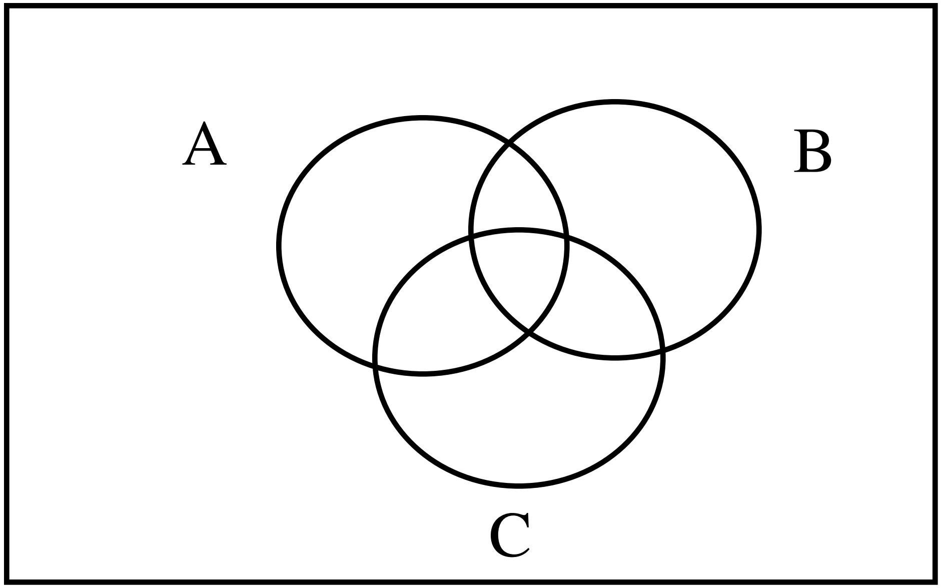
Introduction to Reliability Engineering

Venn Diagrams

- A Venn diagram is a pictorial representation of the relationship between sets. To draw a Venn diagram we start with the following picture (for three sets):



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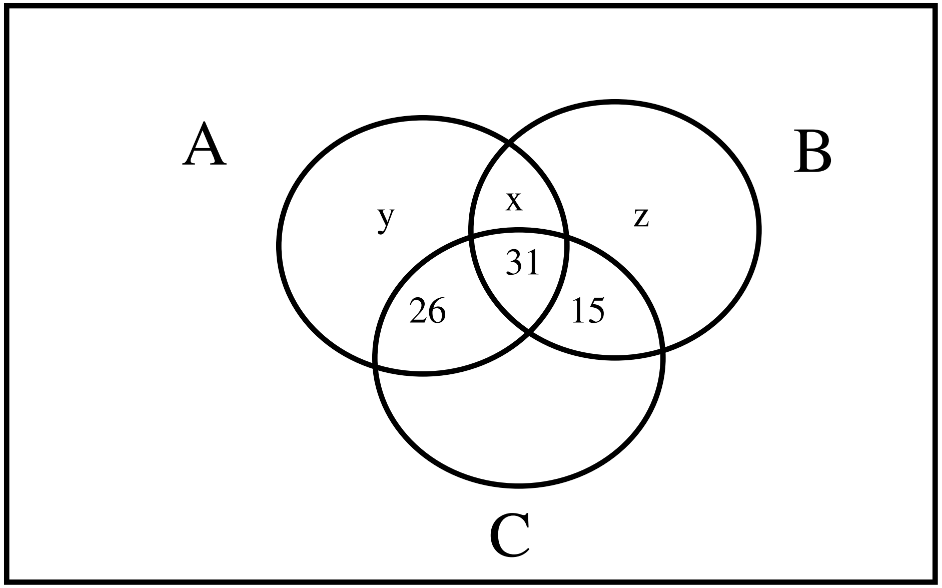
Venn Diagram…an example

- In a survey carried out in a school snack shop, the following results were obtained. Of 100 boys questioned, 78 liked sweets, 74 ice cream, 53 cake, 57 liked both sweets and ice cream. 46 liked both sweets and cake while only 31 boys liked all three. If all the boys interviewed liked at least one item, how many boys liked both ice cream and cake?

Venn Diagram…an example…solution

- Let A= set of boys who like ice cream
- Let B= set of boys who like cake
- Let C= set of boys who like sweets



1. (78-26-31-15)=6

2. Solve: $\begin{cases} 26+31+x+y=74 \\ 15+31+x+z=53 \\ 26+31+15+6+x+y+z=100 \end{cases}$

3. z=5, x=2, y=15

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Probability History lesson problem solved...

What’s likelier:
Rolling at least one 6 in four throws of a single die, or rolling at least one DOUBLE 6 in 24 throws of a pair of dice?

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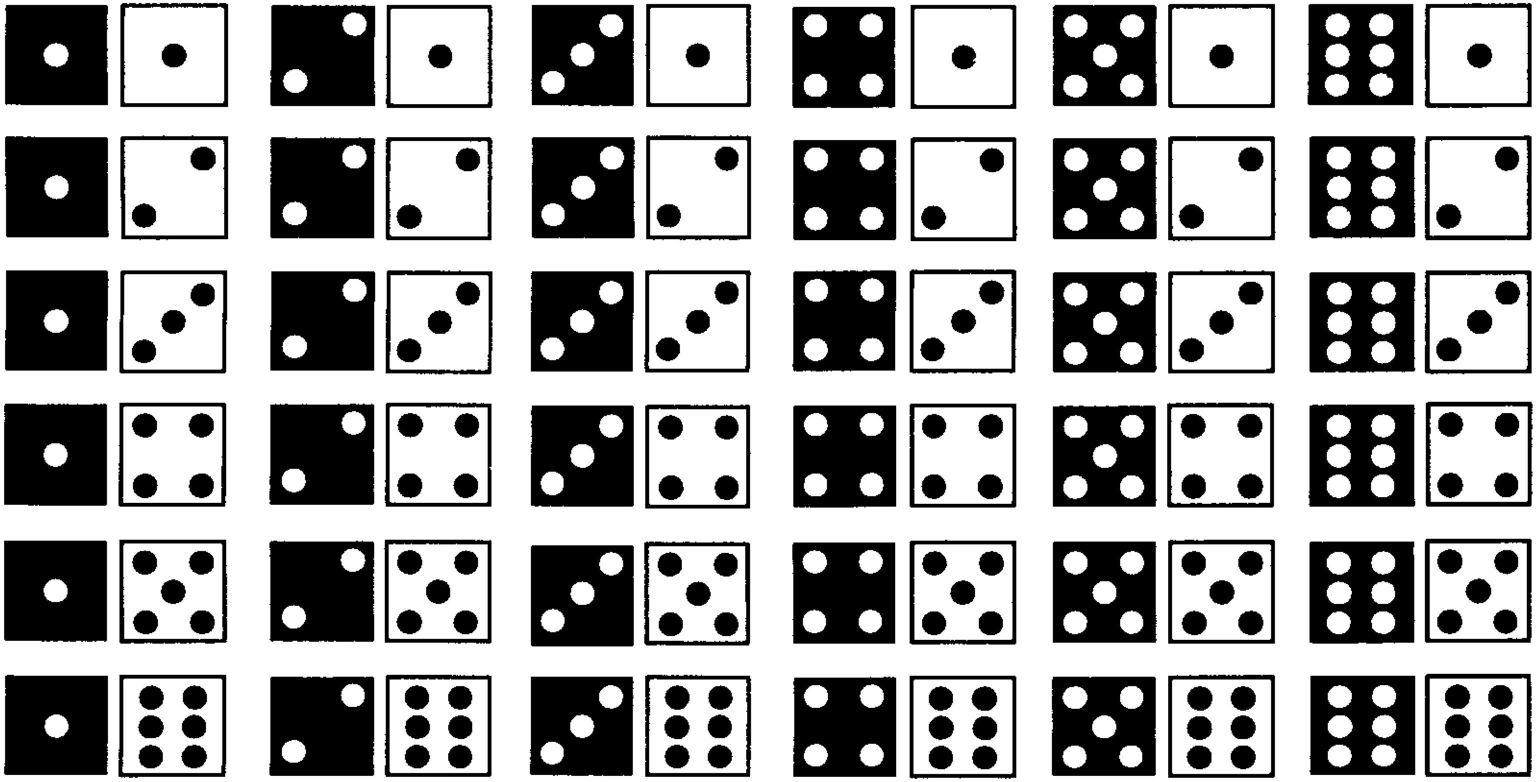
Introduction to Reliability Engineering

Sample space for a pair of dice



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Probability History lesson...continued

Let E=getting at least one 6 in four rolls of a single die.

Then P(E)=1-P(Not E)
=1-(P(Not on 1st))(P(Not on 2nd))(P(Not on 3rd))(P(Not on 4th))
=1-(5/6)(5/6)(5/6)(5/6)=.518

Let F=getting at least one DOUBLE 6 in 24 throws of a pair of dice.

Then P(F)=1-P(Not F)=1-((P(Not on ith))^{24}
=1-(35/36)^{24}=.491

A real world problem in conditional probability--Inspection technique

- A test was done on ultrasonic inspection kits to determine how effective they are in discovering microscopic cracks in parts. When a crack was present, the equipment signaled a crack 98% of the time. There was a “false alarm” on 3% of the parts which had no cracks. Suppose five percent of all the parts have a crack in them. If the percentages in the test can be assumed to be the true probabilities, find the probability that a part is really bad when a kit signals a crack.

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Ultrasonic inspection capability (continued)

Let A= event that the part has a crack

Let B= event ultrasonic inspection indicates a part has a crack.

Now,

P(A)=.05 (5 parts in 100 have a crack)

P(B|A)=.98 (Probability of a positive test, given a crack, is .98)

P(B| NOT A)=.03 (Probability of a false positive, given no crack, is .03)

P(A|B)=? (Probability of having a crack, given a positive test)

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Ultrasonic inspection capability (continued)

	A	Not A	SUM
B	$P(A \cap B)$	$P(\text{Not } A \cap B)$	$P(B)$
Not B	$P(A \cap \text{Not } B)$	$P(\text{Not } A \cap \text{Not } B)$	$P(\text{Not } B)$
	.05	.95	1

Now, $P(A \cap B)=P(B|A)P(A)=(0.98)(0.05)=.049$

$P(\text{Not } A \cap B)=P(B|\text{Not } A)P(\text{Not } A)=(0.03)(0.95)=0.0285$

So,

	A	Not A	SUM
B	.049	.0285	.0775
Not B	$P(A \cap \text{Not } B)$	$P(\text{Not } A \cap \text{Not } B)$	$P(\text{Not } B)$
	.05	.95	1

and,

	A	Not A	SUM
B	.049	.0285	.0775
Not B	.001	.9215	.9225
	.05	.95	1.0

Ultrasonic inspection capability, answer

So, the probability that a part is **really bad**, given an ultrasonic inspection indicates a crack:

$$P(A|B)=P(A \cap B)/P(B)=.049/.0775=.6322$$

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Homework Probability problems....

- Select a Card at random from a deck of 52 cards. What is the probability the card is a spade face card?
- Three light bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that 1) none is defective, 2) exactly one is defective, 3) at least one is defective.
- A class contains 10 men and 20 women of which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.
- A pair of fair dice is thrown. Find the probability p that the sum is 10 or greater if 1) a 5 appears on the 1st die, 2) a 5 appears on at least one die.
- An urn contains 7 red marbles and 3 white marbles. Three marbles are drawn from the urn one after the other. Find the probability that the first two are red and the third is white.

Introduction to Reliability Engineering

Month	**Day**	**Time**	**Title**
February	9	Noon-2PM	Class Introduction-Overview
February	23	Noon-2PM	Probability (Hint: Reliability is Applied Probability)-Part 1
➡ March	8	Noon-2PM	Probability (Hint: Reliability is Applied Probability)-Part 2
March	22	Noon-2PM	Discrete and Continuous distributions-Part 1
April	12	Noon-2PM	Discrete and Continuous distributions-Part 2
April	26	Noon-2PM	Weibull Distribution -Part 1
May	10	Noon-2PM	Weibull Distribution -Part 2
May	24	Noon-2PM	Reliability Modeling -- parallel, series, redundant, standby systems-Part 1
June	14	Noon-2PM	Reliability Modeling -- parallel, series, redundant, standby systems-Part 2
June	28	Noon-2PM	FMEA-Part 1
July	12	Noon-2PM	FMEA Part 2
July	26	Noon-2PM	Reliability Testing -Part 1
August	9	Noon-2PM	Reliability Testing -Part 2
August	23	Noon-2PM	Reliability Testing -Part 3
September	13	Noon-2PM	Reliability Allocations and Predictions, Reliability Growth-Part 1
September	27	Noon-2PM	Reliability Allocations and Predictions, Reliability Growth-Part 2
October	11	Noon-2PM	System Safety Analysis and link to Reliability-Part 1
October	25	Noon-2PM	System Safety Analysis and link to Reliability-Part 2
November	8	Noon-2PM	Maintainability & Human Reliability-Part 1
November	22	Noon-2PM	Maintainability & Human Reliability-Part 2
December	13	Noon-2PM	Other Topics (TBD)

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Homework Probability problems....

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- An urn contains 7 red marbles and 3 white marbles. Three marbles are drawn from the urn one after the other. Find the probability that the first two are red and the third is white.

Probability homework solutions

- Select a Card at random from a deck of 52 cards. What is the probability the card is a spade face card?

- Let A= {the card is a spade}
- B= {the card is a face card}
- P(A)=13/52
- P(B)=12/52
- $P(A \cap B)=3/52$

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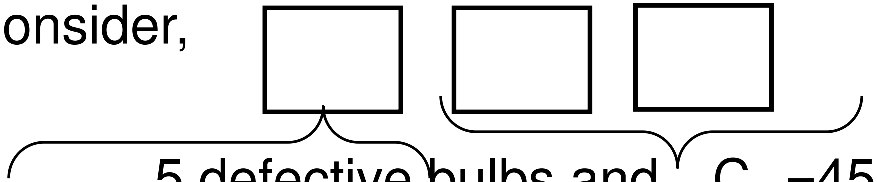
Probability Homework solutions...

- Three light bulbs are chosen at random from 15 bulbs of which 5 are defective. Find the probability that 1) none is defective, 2) exactly one is defective, 3) at least one is defective.
- There are ${}_{15}C_3$ =455 ways to choose 3 bulbs from the 15.

1) There are 15-5=10 nondefective bulbs, and ${}_{10}C_3$ =120 ways to choose 3 nondefective bulbs. So, P(none defective)=120/455

2) Consider,



- 5 defective bulbs and ${}_{10}C_2$ =45 different pairs of nondefective bulbs. Therefore there are 5 x 45=225 ways to choose 3 bulbs of which one is defective. P(one defective)=225/455
- 3) P(at least 1 defective)=1-P(none defective)=1-120/455=335/455
-

Introduction to Reliability Engineering

Probability Homework solutions.....

- A class contains 10 men and 20 women of which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.

- Let A={person is a man}
- B={person has brown eyes}

- P(A)=10/30=1/3
- P(B)=15/30=1/2
- $P(A \cap B)=5/30=1/6$

- $P(AUB)=P(A)+P(B)-P(A \cap B)=1/3+1/2+-1/6=2/3$

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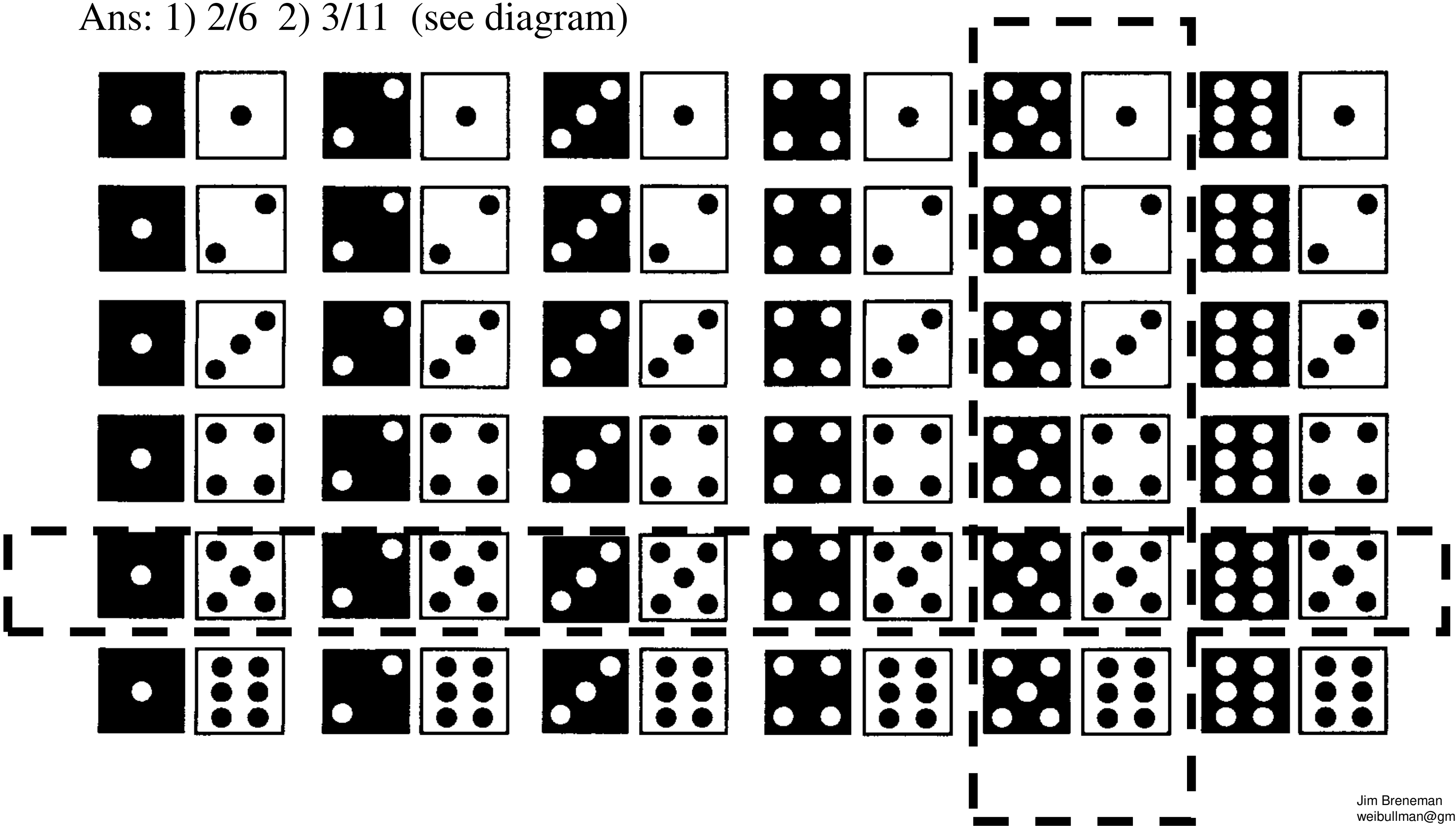
Probability Homework solutions

- A pair of fair dice is thrown. Find the probability p that the sum is 10 or greater if 1) a 5 appears on the 1st die, 2) a 5 appears on at least one die.

Ans: 1) 2/6 2) 3/11 (see diagram)



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Probability Homework solutions

- An urn contains 7 red marbles and 3 white marbles. Three marbles are drawn from the urn one after the other. Find the probability that the first two are red and the third is white.

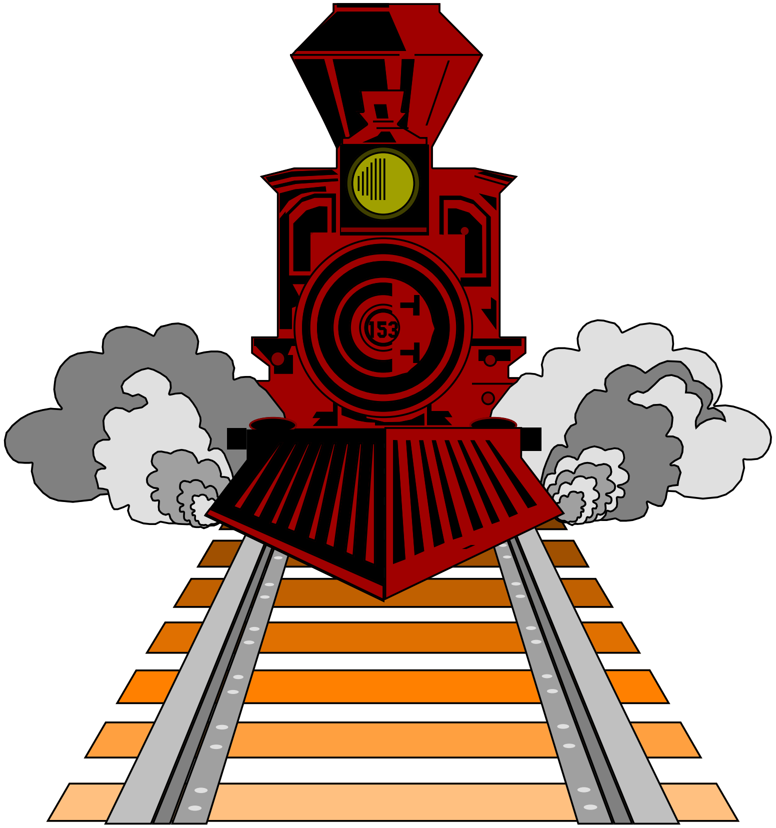
- 10 total marbles.
- P(1st marble is red)=7/10
- P(2nd marble is red| 1st marble is red)=6/9
- P(3rd marble is white| 1st two red)=3/8

- P(1st two red and 3rd white)= 7/10 x 6/9 x 3/8 = 7/40

Introduction to Reliability Engineering

** Probability*

Let's continue counting!



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Use of Geometric Progression in Probability

If $S = a + ar + ar^2 + ar^3 + \dots \infty$

$$S = \frac{a}{1 - r},$$

for $|r| < 1$, a is the 1st term,

r is the common ratio

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Use of Geometric Progression… an Example

Three people: A, B, C in that order, throw a tetrahedral (four-sided) die. The first one to throw a "4" wins. The game is continued indefinitely until someone wins. Find the probability that

1. A wins
2. B wins
3. C wins

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Use of Geometric Progression… an Example

1. Let A_i be the event 'A wins on his i^{th} throw'

   $P(A_1) = 1/4$, $P(\bar{A}_1) = 3/4$

   $P(A_2) = P(\bar{A}_1 \bar{B}_1 \bar{C}_1 A_2) = (3/4)^3(1/4)$

   $P(A_3) = P(\bar{A}_1 \bar{B}_1 \bar{C}_1 \bar{A}_2 \bar{B}_2 \bar{C}_2 A_3) = (3/4)^6(1/4)$

   …

Therefore, $P(\text{A wins}) = 1/4 + (3/4)^3(1/4) + (3/4)^6(1/4) + \ldots$

$= 1/4\,(1 + (3/4)^3 + (3/4)^6 + \ldots)$

$= 1/4\ S = 1/4\,(1/\,(1-(3/4)^3)) = 16/37$

Introduction to Reliability Engineering

Use of Geometric Progression… an Example

2. Let B_i be the event ‘B wins on his i^{th} throw’

$$P(B_1) = P(A_1\ \bar{B}_1) = (3/4)(1/4)$$

$$P(B_2) = P(\bar{A}_1\ \bar{B}_1\ \bar{C}_1\ \bar{A}_2\ B_2) = (3/4)^4(1/4)$$

$$P(B_3) = P(\bar{A}_1\ \bar{B}_1\ \bar{C}_1\ \bar{A}_2\ \bar{B}_2\ \bar{C}_2\ \bar{A}_3\ B_3) = (3/4)^7(1/4)$$

…

Therefore, $P(B \text{ wins}) = (1/4)(3/4) + (3/4)^4(1/4) + (3/4)^7(1/4) + \ldots$

$$= (1/4)(3/4)\ (1 + (3/4)^3 + (3/4)^6 + \ldots\ )$$

$$= (1/4)(3/4)\ S = (1/4)(3/4)\ (1/\ (1-(3/4)^3)) = 12/37$$

Use of Geometric Progression… an Example

3. Therefore, P(C wins)= 1 - P(A wins) - P(B wins)

= 1 - (16/37) - (12/37)

= 9/37

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Arrangements

The number of ways of arranging *n* unlike objects in a line is *n!*[*]

For example, consider the letters A,B,C,D

Therefore, the number of ways of arranging the 4 letters is 4!=24

Checking this, the arrangements are:

ABCD ABDC ACBD ACDB ADCB ADBC
BCDA BCAD BDAC BDCA BACD BADC
CDBA CDAB CABD CADB CBAD CBDA
DABC DACB DBCA DBAC DCAB DCBA

[*] where N!=1 x 2 x 3 x ... x (N-1) x N
and 1! = 1, 0!=1

Arrangements 2

The number of ways of arranging in a line *n* objects, of which *p* are alike, is *n!/p!.*

For example, given the letters A,A,A,D; the 24 arrangements listed previously reduce to AAAD AADA ADAA DAAA or, using the formula *4!/3!* = 4.

The number of ways of arranging in a line *n* objects of which *p* of one type are alike, *q* of a second type are alike, *r* of a third type are alike, and so on, is *n!/(p!q!r!…)*

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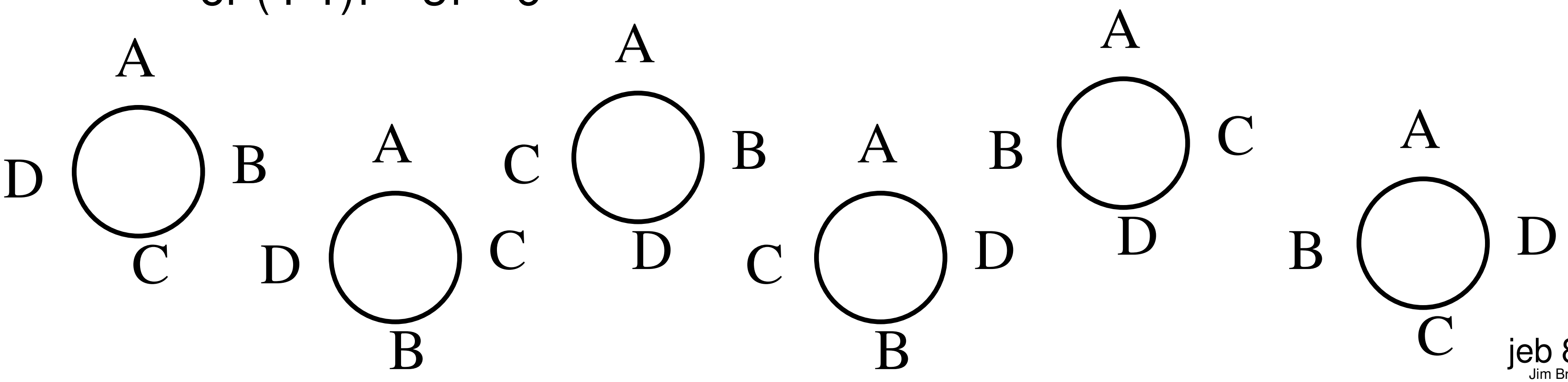
Arrangements 3

The number of ways of arranging *n* unlike objects in a ring when clockwise and counterclockwise arrangements are different is *(n-1)!*.

For example, consider 4 people A,B,C,D, who are seated at a round table. To find the number of different arrangements, we fix A and then consider the number of ways of arranging B,C,D,

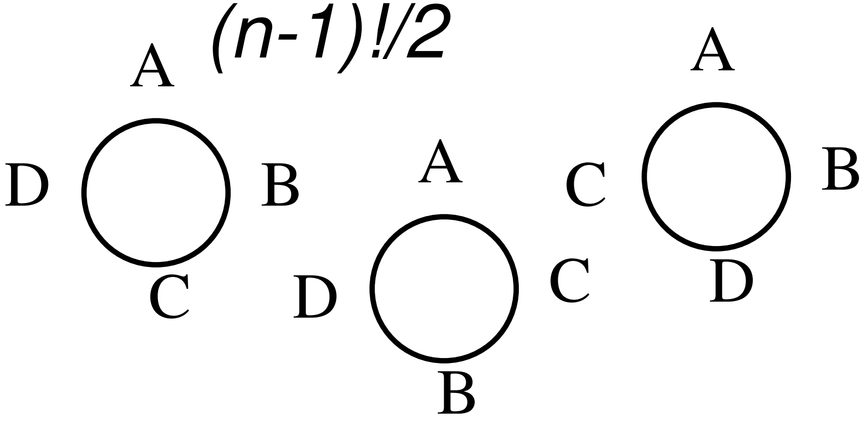
or (4-1)! = 3! = 6



Arrangements 3 (continued)

The number of ways of arranging *n* unlike objects in a ring, when clockwise and counterclockwise arrangements are the same, is

$$(n-1)!/2$$



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Arrangements… an Example

Given a “rainbow” wheel of 36 blades, with 4 different vendors
(9 blades each):

1. How many ways can these 36 blades be arranged in this wheel?

   (36-1)!/2=35!/2 = 5,166,573,983,193,072,464,833,325,668,761,600,000,000 ways

2. How many ways can these 36 blades be arranged in this wheel, considering the 4 vendors?

   36!/(9!9!9!8!)=193,074,770,396,387,880,000 ways

Thank you Mathematica™

jeb 91
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Permutations

How many ways can 5 cards be selected randomly from a deck of 52 cards when order is important?

A K Q J 10	Permutation
A Q K J 10	Different events

A permutation of N different objects taken R at a time is an arrangement of R out of the N objects with attention given to the order of arrangement.

Written:

$$_{N}P_{R}=\frac{N!}{(N-R)!}$$

where $N!=1 \times 2 \times 3 \times \ldots \times N$

$1! = 1 \quad 0!=1$

e.g.

$$_{52}P_{5}=\frac{52!}{47!}=\frac{52 \times 51 \times 50 \times 49 \times 48 \times 47!}{47!}=311,875,200$$

Combinations

How many ways can 5 cards be selected randomly from a deck of 52 cards when order is not important?

A K Q J 10 Combination
A Q K J 10 Considered as the same event

A combination of N different objects taken R at a time is a selection of R out of the N objects with NO attention given to the order of arrangement.

Written:

$$ {}_{N}C_{R} = \frac{N!}{R\,!(N-R)!} $$

where $N! = 1 \times 2 \times 3 \times \ldots \times N$

$1! = 1$

e.g. $$ {}_{52}C_{5} = \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47!}{(1 \times 2 \times 3 \times 4 \times 5\,)47!} = 2{,}598{,}960 $$

Using Permutations

- How many different numbers of 3 digits can be formed from the numbers 1,2,3,4,5.

  (a) If repetitions are allowed? $\boxed{5} * \boxed{5} * \boxed{5} = 125$

  (b) If repetitions are not allowed? $\boxed{5} * \boxed{4} * \boxed{3} =60$

- Calculate the number of permutations of the letters a,b,c,d taken two at a time, taken four at a time. ${}_4P_2 \quad \& \quad {}_4P_4$

- How many permutations of the seven letters of the word *algebra*?

$$\text{Permutations of "algebra"} = \frac{7!}{2!} = \frac{7*6*5*4*3*2!}{2!} = 2520$$

- In how many ways may a party of four women and four men be seated at a round table if the women and men are to occupy alternate seats?

$$\textit{Ways to seat } 4 \textit{ women} \,\&\, 4 \textit{ men at round table} = (4\text{-}1)!*4!=144$$

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Using Combinations

- How many different sums of money can be obtained by choosing two coins from a box containing a penny, a nickel, a dime, a quarter, and a half dollar?

$$Different\ sums\ money = {}_5C_2 = 10$$

- How many baseball teams of nine members can be chosen from among twelve boys, without regard to the position played by each member?

$$Different\ teams = {}_{12}C_9 = \frac{12!}{9!3!} = \frac{12*11*10}{3*2*1} = 220$$

- How many "words" each consisting of two vowels and three consonants, can be formed from the letters of the word *integral*?

$$\#\ "Words" = {}_3C_2 * {}_5C_3 * {}_5P_5 = 3*10*5! = 3600$$

- How many different bridge hands are there?

$$Different\ bridge\ hands = {}_{52}C_{13} = 635,013,559,600$$

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A Perfect Bridge Deal?

While traveling out West a few years ago I read in the paper that a perfect Bridge deal was reported to have been dealt in Las Vegas. Does that seem likely?

$$\text{Prob (Perfect Bridge deal)} = \frac{4}{{}_{52}C_{13}} \times \frac{3}{{}_{39}C_{13}} \times \frac{2}{{}_{26}C_{13}} \times \frac{1}{{}_{13}C_{13}}$$

$$= 3.85 \times 10^{-28}$$

So, on average, we expect a perfect deal once every 100,000 Trillion years @ billion hands/day.

==> <u>Not random!!</u>   stacked deck!!

Birthday problem.....

How many people in a group to have a 50:50 chance (probability $\geq .5$) that at least two have the same birthday?

- Prob (No match | r people) = $\dfrac{365 \text{ x } 364 \text{ x } \ldots \text{ x}(365-r+1)}{365^r}$
- Prob (Match| r people)=1-Prob(No match|r people)

r	5	10	20	23	30	40	60
P(Match)	.027	.117	.411	.507	.706	.891	.994

Look at this carefully... does it look reasonable?

* Not accounting for a leap year

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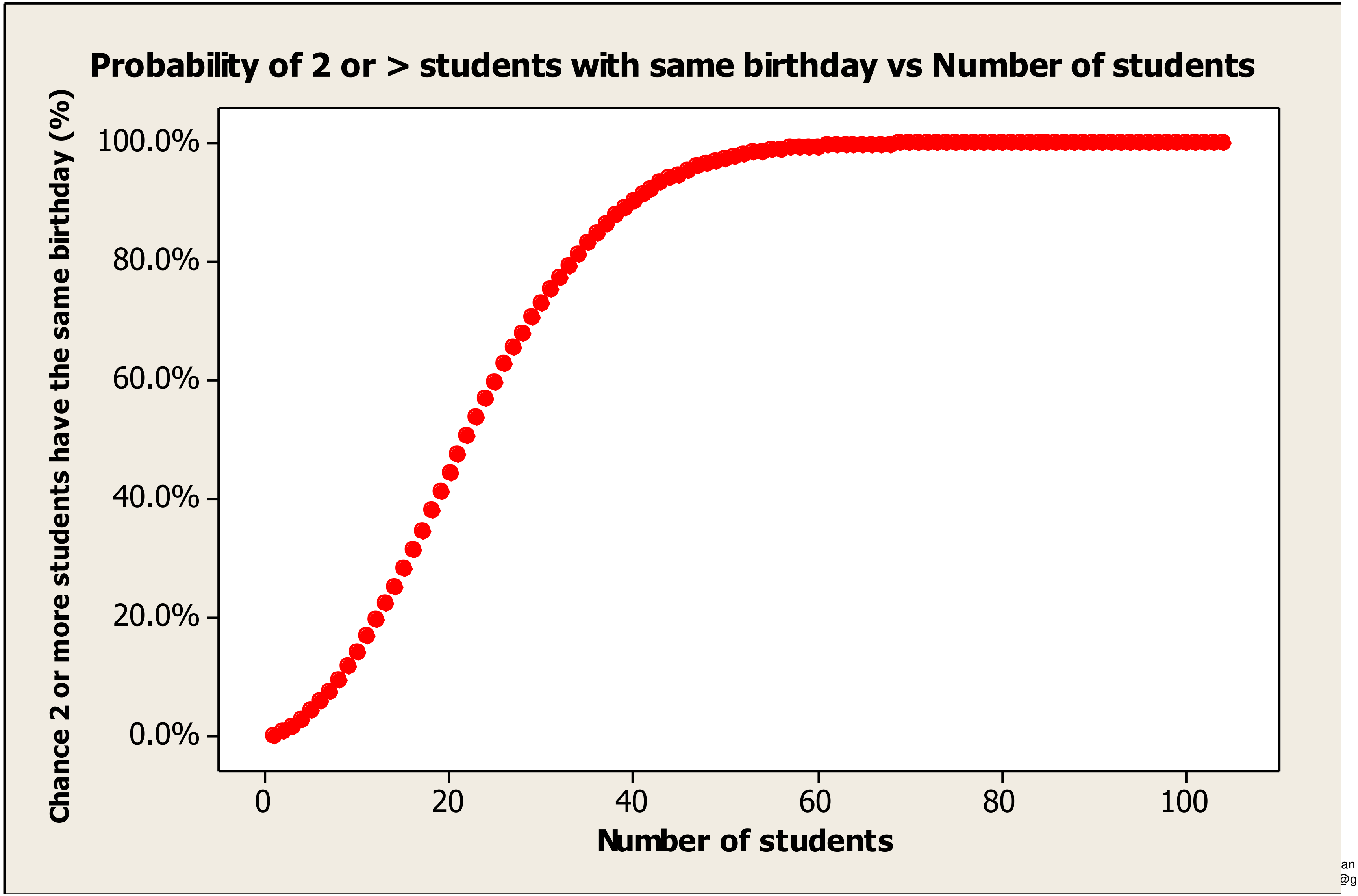
Birthday problem



Introduction to Reliability Engineering

Ranking of poker hands and their probability of occurrence.

Hand	# of ways	Probability
Straight flush	40	0.000015
Four of a kind	624	0.000240
Full house	3744	0.001441
Flush	5108	0.001965
Straight	10200	0.003925
Three of a kind	54912	0.021129
Two pair	123552	0.047539
One Pair	1098240	0.422569
No pair Hand	1302540	0.501177
Total hands	2598960	1

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Solutions to Poker probabilities

Straight flush:

A 2 3 4 5 6 7 8 9 10 J Q K A...10 ways, 4 suits: 4 x 10=40 ways

Four of a Kind:

Four matching are one of 13 "denominations", 5th card from remaining 48: 13 x 48=624 ways

Full house (3 of a kind with 2 of a kind):

13 denominations for ${}_4C_3$ and 12 denominations for ${}_4C_2$

13 x 4 x 12 x 6=3744 ways

Flush:

One of four suits, 5 denominations from 13 possible or

4 x ${}_{13}C_5$ =5148 -{straight flushes}=5148-40=5140

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Solutions to Poker probabilities (continued)

Straight:

A 2 3 4 5 6 7 8 9 10 J Q K A (10 ways), each card can be from one of the suits: 10 x 4 x 4 x 4 x 4 x 4=10240-{straight flushes}

=10240-40=10200 ways

Three of a kind:

Any three of the 4 of one kind, 13 denominations, the remaining two cards different from the 3:

$13 \times {}_4C_3 \times {}_{48}C_2$ = 58656-{Full house}=58656-3744=54912 ways

Two pair:

${}_{13}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{44}C_1$ =123552 ways

One pair:

$13 \times {}_4C_2 \times {}_{12}C_3 \times 4 \times 4 \times 4$= 1098240

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Odds

Probabilities are sometimes stated in terms of *odds*.

If a player bets on a specific slot at roulette, his probability of winning is 1/38, and his probability of losing is 37/38:

The *odds against* an event is the ratio of the probability that the event does not occur to the probability that the event does occur,

$$\text{Odds against winning} = \frac{\text{probability of not winning}}{\text{probability of winning}} = \frac{37/38}{1/38} = \frac{37}{1}$$

or 37 to 1 (sometimes written 37:1).

$$\text{The odds in favor of winning} = \frac{1/38}{37/38} = \frac{1}{37} \text{ or 1 to 37 (1:37).}$$

Probability of a Lottery win

The SC state run Educational "Powerball" lottery pays off for 3, 4, 5 numbers picked correctly as well as for 3,4, or 5 picked correctly and a Powerball number picked correctly.

The numbers chosen for the first 5 come from the integers 1,2,3,4,5,...,59

The Powerball number comes from the integers 1,2,3,…,39

What is the probability of all 6 being drawn correctly?

The number of ways to draw 5 numbers out of 59=${}_{59}C_5$=5,006,386

Therefore, P(all 5 correct)=1/5,006,386

The probability of getting the 5 "white" numbers correct AND the Powerball=(1/5,006,386)*(1/39)
=1/195,249,054

The number of ways to draw 4 numbers out of 5 correctly=
${}_5C_4 \times {}_{54}C_1$ = 270, therefore P(4 out of 5)=270/5,006,386

The number of ways to draw 3 numbers out of 5 correctly=
${}_5C_3 \times {}_{54}C_2$ = 14310, therefore P(3 out of 5)=14310/5,006,386

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Probability of a Powerball win

The CT Powerball lottery pays off if the 1st 5 numbers, plus a "powerball" are selected correctly.

The 1st 5 numbers chosen come from the integers 1,2,3,4,5,...49

The 6th "powerball" number comes from the integers 1,2,3,…,42

What is the probability of all 6 being drawn correctly?

The number of ways to draw 5 numbers out of 49=${}_{49}C_5$=1,906,884

Then, a "powerball" number is chosen from 1,…42;

Therefore, P(all 6 correct)= 1,906,884 * 42 = 80,089,128

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Introduction to Reliability Engineering

What are the odds?... 1 chance in

Winning Powerball lottery	80,089,128
Struck & killed by falling aircraft	25,000,000
Winning CT lottery	13,983,816
Killed by lightning	1,603,250
Dying from venomous bite/sting	1,159,364
Freezing to death	780,938
Struck by lightening	576,000
American male dating a Supermodel	88,000
Being murdered (female)	79,365
Being blackmailed	52,632
Being murdered (male)	45,249
Being Kidnapped	33,223
Having malaria	21,739
Being in prison (female)	6757
Trifecta (13-horse race)	1716
Being in prison (male)	396
Having car stolen	142
Roulette number coming up	37
Pulling an ace out of a deck of cards	13
Hiring a sleazy lawyer	8
Rolling a 7 or 11 in craps	4.5
Dying from heart disease	4.0

Household with couple and no children	3.0
Marriage ending in divorce	2.3
Married couple aged within two years of each other	2.2
Bride being older than 26 at first wedding	2.0

- Dying in a sporting accident this year:
  - Mountain climbing -- 1 in 167 (1 in 5 lifetime)
  - Hang Gliding -- 1 in 4,444
  - Skydiving -- 1 in 86,000 jumps
  - Auto Racing -- 1 in 1000 to 1 in 5000 depending on type
  - Motorcycling -- 1 in 1000
  - Running -- 1 in 10,000
  - Boating and Swimming -- 1 in 36,000
  - Playing football -- 1 in 57,000
  - Bicycling -- 1 in 130,000

safest sports: Badminton and ping pong

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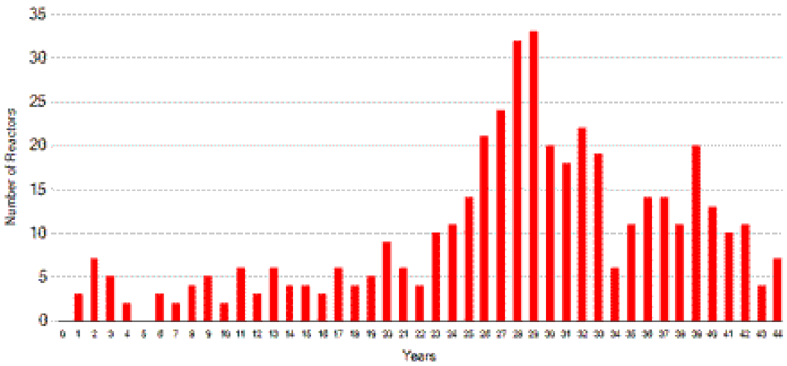
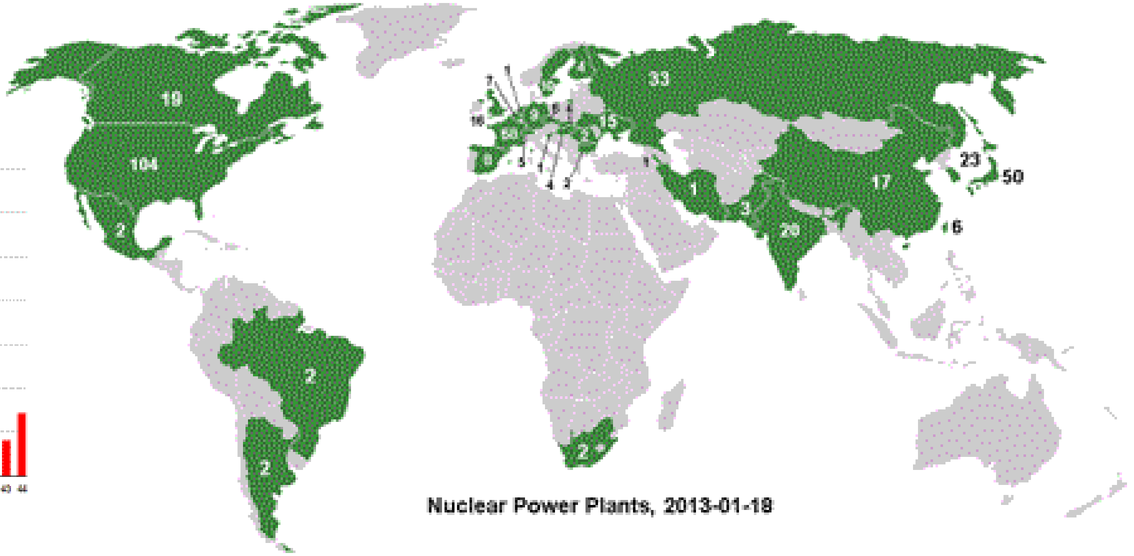
Nuclear reactor safety

- The NRC currently requires only that the builder prove that the chance of a meltdown for a given reactor in a given year is < 1/10,000.
- How does that compare to history?





Nuclear Power Plants, 2013-01-18

- Calculations:
  - 427 nuclear power plant units in 31 countries. With 73 more under construction.*
  - 5 serious commercial incidents since 1951(INES 5 or>)**
  - So,

  Probability of a “serious nuclear power plant incident” ~5/(Nuclear operating years)=~5/18,826* or 2.7/10,000; which > the 2X stated NRC goal.

  According to: http://www.euronuclear.org/info/encyclopedia/n/nuclear-power-plant-world-wide.htm
  *As of July 1, 2013
  (the total electricity production since 1951 amounts to 64,600 billion kWh. The cumulative operating experience amounted to 18,826 years by June 2013.)
  ** Chernobyl,Ukraine(1986), Kyshtym ,Russia(1957), Windscale, UK, (1957); Three Mile Island (1979), Fukushima (2011)
  INES= **I**nternational **N**uclear **E**vent **S**cale

By the way; what we did was a quick “Risk analysis”

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Introduction to Reliability Engineering

What are the odds?... 1 chance in…..

Winning Powerball lottery	146,107,962
Struck & killed by falling aircraft	~25,000,000
Winning CT lottery	7,059,052
Killed by lightning	1,603,250
Dying from venomous bite/sting	1,159,364
Freezing to death	780,938
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Having car stolen	142
Roulette number coming up	37
Pulling an ace out of a deck of cards	13
Hiring a sleazy lawyer	8
Rolling a 7 or 11 in craps	4.75
Dying from heart disease	4.0

Household with couple and no children	3.0
Marriage ending in divorce	2.3
Married couple aged within two years of each other	2.2
Bride being older than 26 at first wedding	2.0
Passenger Surviving a commercial airplane ditching	1.2

- Dying in a sporting accident this year:
  - Mountain climbing -- 1 in 167 (1 in 5 lifetime)
  - Hang Gliding -- 1 in 4,444
  - Skydiving -- 1 in 86,000 jumps
  - Auto Racing -- 1 in 1000 to 1 in 5000 depending on type
  - Motorcycling -- 1 in 1000
  - Running -- 1 in 10,000
  - Boating and Swimming -- 1 in 36,000
  - Playing football -- 1 in 57,000
  - Bicycling -- 1 in 130,000

  safest sports: Badminton and ping pong

Caveat: as of 2008