Bayesian Multimodal Models for Risk Analyses of Low-Probability High-Consequence Events

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Objective

- Bayesian hierarchical modeling method for quantification of uncertainty in multimodal models for estimating failure probabilities in rare hazards
- Application: Prediction of damage and loss probability to built structures due to wind-storms, and assess effects of uncertainty, for insurance underwriting and local hazard mitigation

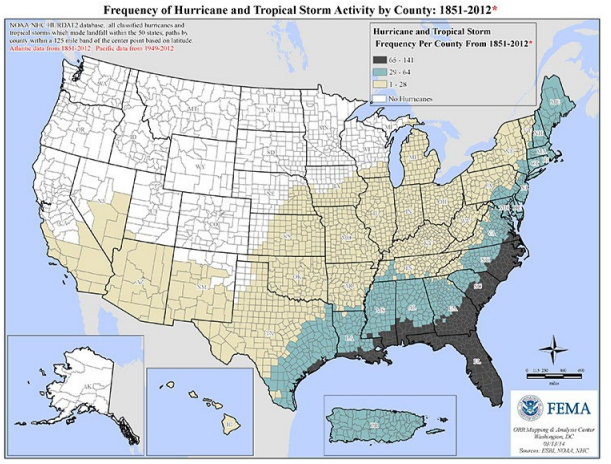
Outline

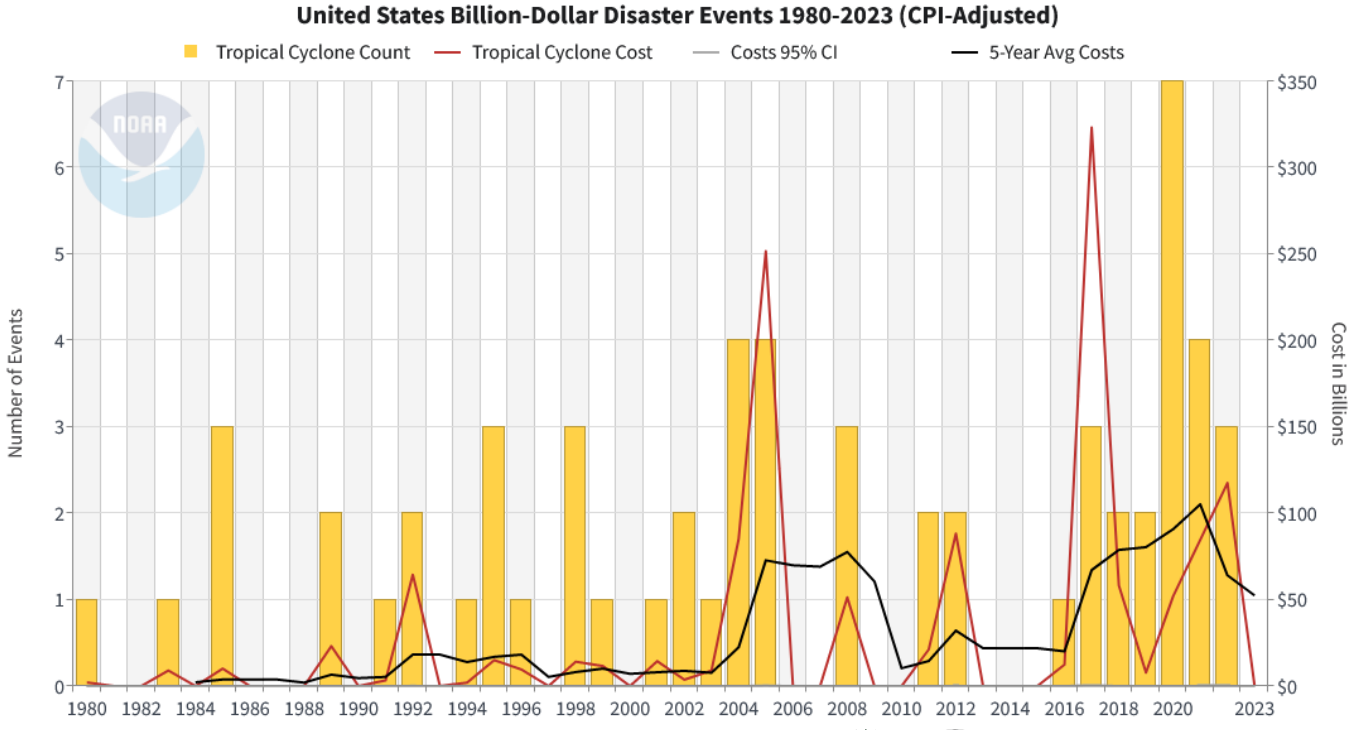
- Introduction - Wind Hazard in the US
- Challenges in predicting losses of wood-frame buildings under hurricane winds
- Proposed Method - Bayesian model updating for predicting hurricane loss
- Case study: Effects of uncertainty in residential building loss estimation
- Conclusions

Introduction - Wind hazard in the US

- Tropical storms and hurricanes accounted for $3.8B per year damage from 2010 to 2020 (NWS, 2020).
- Insured value of properties continues to rise: In coastal counties, $10 trillion are at risk of hurricane force winds (Core Logic, 2022)
- Hurricane Ian caused $60B in insured losses in 2022, leading to six insurers becoming insolvent (III, 2023)

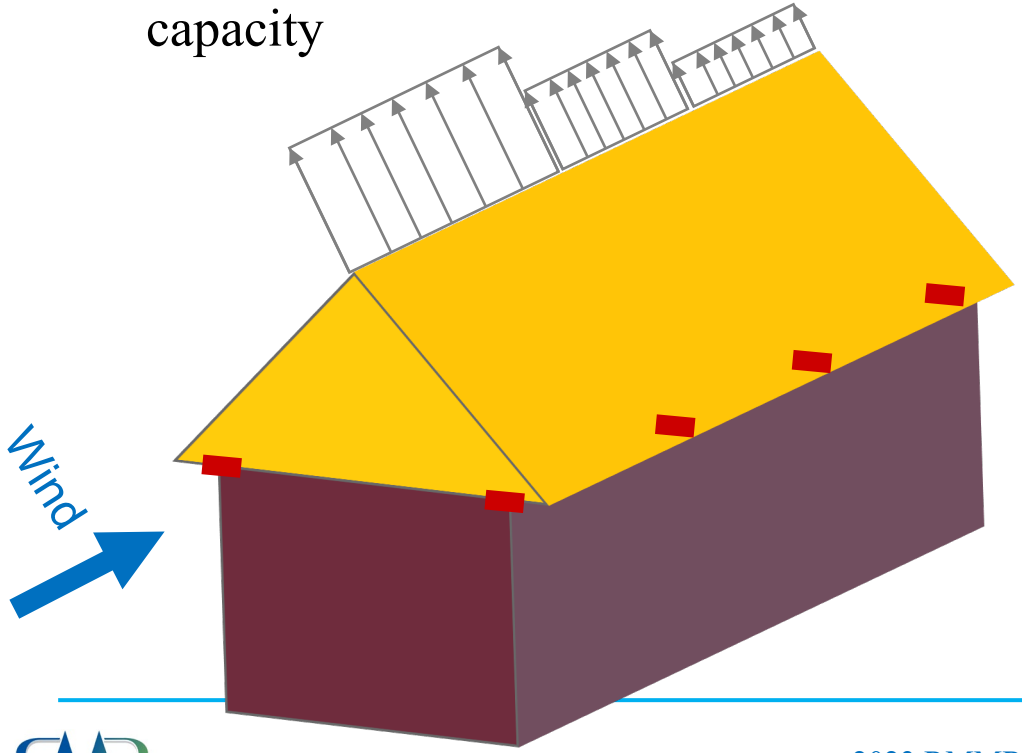
coastal counties in S&SE US: largest hurricane activity





Challenges in predicting losses of wood-frame buildings under hurricane winds

- Damages to residential wood-framed buildings during hurricanes constitute a major economic loss in coastal communities.
- Failure of roof-to-wall connections (leads to water damage to the interior of the building) has a dominant impact in the overall loss
- Failure of a RTW connection occurs when wind load exceeds component capacity

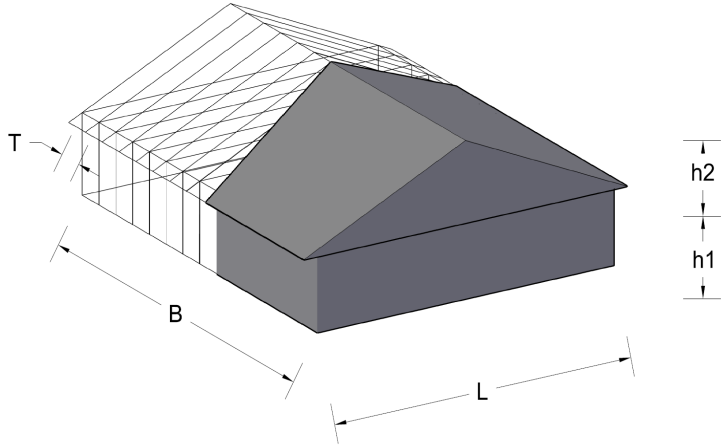


Roof-to-wall connections

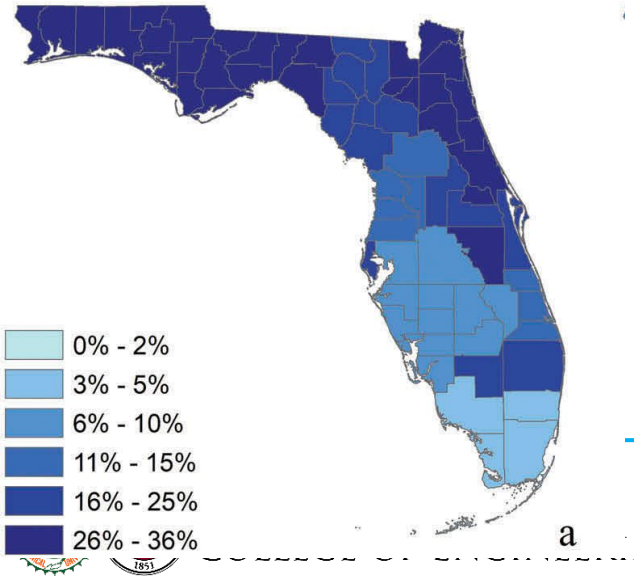
Challenges in predicting losses of wood-frame buildings under hurricane winds

- Main challenges in predicting failures against extreme hurricane events from available loss data
  - Loss data come in the form of either insurance claims or field surveys after extreme (low-probability) events
  - Loss data grouped by structural characteristics (age, number of floors) and geographic location (wind speed zone): highly sparse, resulting in large uncertainties in performance predictions.

One-story wood frame model building



Percentage in FL of buildings built in 1990s



Proposed Method

- A Bayesian hierarchical regression model to fuse capacity predictions of an analytical model of capacity and loss data from field surveys
  - A physics-based analytical model is used to formulate the prior distribution of the model parameters
  - Develop the posterior predictive distribution of failure probabilities (which involves integrating a fragility function weighted by the wind speed probability distribution) of a RTW connection
  - Propagate uncertainty of failure probability of individual connections to estimate aggregate losses in a building stock of a geographical region
  - Compare cost effectiveness of various mitigation options (valuable for designing incentives)
  - Account effect of lack of available data in cost-benefit analyses

Proposed Method - Wind fragility

- Probability of failure of a RTW connection under hurricane winds

$$P_f = \int F_v \, p(v) dv$$

  - $p(v)$: PDF of wind speed v. E.g., Weibull distribution fitted to historical wind speed data
  - F_v: Fragility function: conditional probability that wind load exceeds capacity

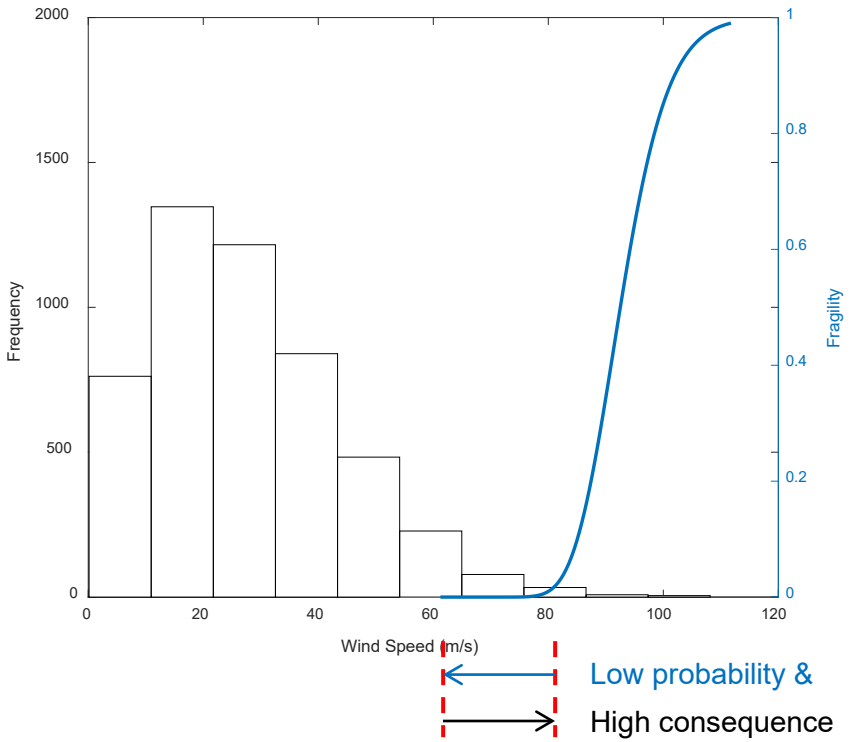
$$F_v = P(D_v \geq C)$$

- Lognormal fragility functions (Rosowsky et al, 2002)

$$F_v = \Phi[\ln(D_v/m)/\zeta]$$

  - m :median capacity,
  - ζ :logarithmic standard deviation of capacity

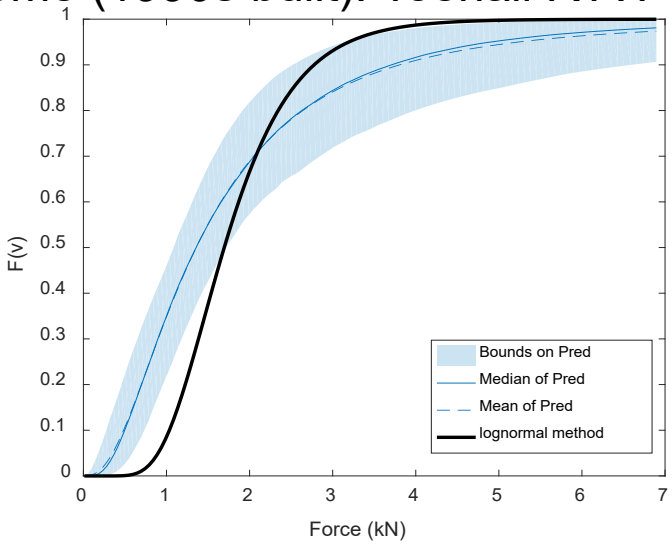
- Both F_v and $p(v)$ estimated from sparse data; Estimation of P_f contains substantial uncertainty
  - Lack of available data of component performance in high wind speed domains, the low-probability high-consequence event



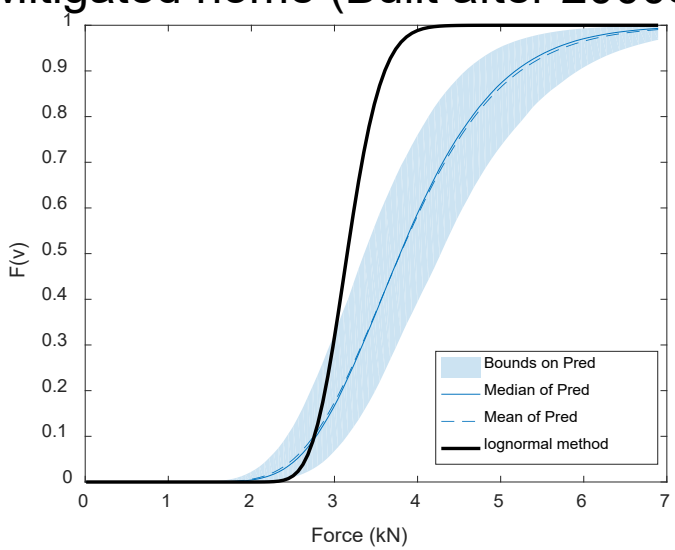
Proposed Method - Cost Benefit Analysis

- Mitigation or hardening of homes is an effective strategy to lessen the damages due to hurricanes: Cost-benefit analysis for mitigation policy making
  - Whether to upgrade your roof or install wind shutters considering risk of a 100-year hurricane?
  - Government incentives for mitigation – roof upgrade tax credit, insurance premium discounts, reduced property taxes

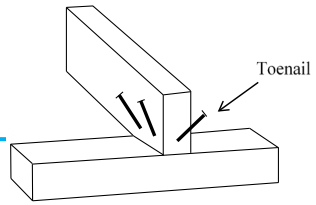
Older home (1990s built): Toenail RTW



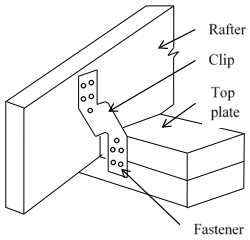
Mitigated home (Built after 2000s): Clip RTW



Toe-nail RTW



Clip RTW



Proposed Method - Cost Benefit Analysis

- From the probability of failure P_f, find the expected aggregate loss in a building inventory
  - Assumptions: probability of the hurricane (wind speed PDF) affecting all buildings throughout the region is equal, the connections (fragility) are identical→ P_f is the same for all houses in the inventory
  - Inventory contains n houses, N connections per house, $M = N \times n$ connections. Number of failed RTW connections,

$$p_{RTW}(w) = \binom{M}{w} P_f^w (1 - P_f)^{M-w}, \qquad \text{for } w = 0,1,\dots,M$$

  - Each house is worth \$100k, loss per house due to a connection failure is 20% of the building value

$$L_0 = 0.20 \times 100{,}000 = \$20{,}000$$

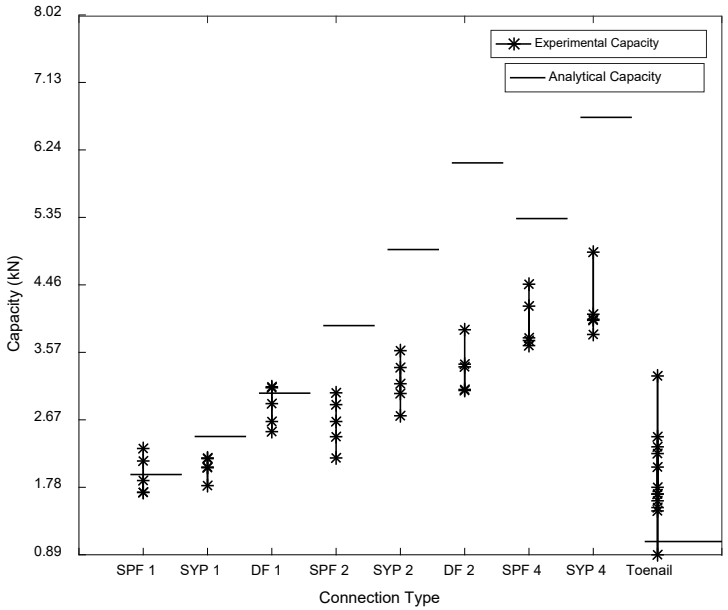
  - Expected aggregate loss, exceedance probability

$$E[L] = L_0 M P_f$$

$$P(L > x) = P\left(w > \frac{x}{L_0}\right) = \sum_{w=\lfloor l/L_0 \rfloor}^{M} p_{RTW}(w)$$

Proposed Method – Physics-based model

- Wind load D_v: closed form expression available for given roof geometry and wind speed
- Analytical capacity model $C_a(x)$:physics-based, expression available
- Experimental capacity data $C_e(x)$: reported in literature for clip and toe-nail connections
  - x: wood type, geometry, number of clips
  - Wood type: Spruce Pine Fir (SPF), Southern Yellow Pine (SYP),Douglas Fir (DF)
  - number of clips: 1, 2 and 4



Proposed Method – Bayesian two-stage regression model to fuse analytical predictions with field data

- Fused capacity model

$$C(\boldsymbol{x}) = \boldsymbol{\beta}'\boldsymbol{x} + \epsilon = \delta_0 + \delta_1 x_1 + \delta_{11} x_1^2 + \delta_2 x_2 + \delta_3 x_3 + \epsilon$$

  - $\boldsymbol{\beta}, \boldsymbol{x}$: model coefficients & features (d dimensional); σ^2: error variance
  - $\epsilon \sim N(0, \sigma^2)$ model error
  - x_1 : number of clips (takes values of 1, 2 and 4); x_2, x_3: the dummy regressors for the wood type
  - $\boldsymbol{C}_a = (c_{a1} \dots c_{an_1})$ analytical capacity predictions for n_1 connection configurations specified by a $n_1 \times d$ matrix $\boldsymbol{X}_1$
  - $\boldsymbol{C}_e = (c_{e1} \dots c_{en_2})$ capacity measurements for n_2 connection configurations specified by a $n_2 \times d$ matrix $\boldsymbol{X}_2$.
- First stage model: *non-informative prior* on the analytical capacity model parameters $p(\boldsymbol{\beta}, \sigma^2) \propto 1/\sigma^2$.
  - Posterior of $\boldsymbol{\beta}$ and σ^2 (Gelman et al., 2014)

$$p(\boldsymbol{\beta}|\sigma^2, \boldsymbol{C}_a) = MVN(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$$
$$p(\sigma^2|\boldsymbol{C}_a) = Inv - \chi^2(\nu_1, s_1^2)$$

- Second stage model: prior = posterior of first stage model
  - Posterior of $\boldsymbol{\beta}$ and σ^2

$$p(\boldsymbol{\beta}|\sigma^2, \boldsymbol{C}_e, \boldsymbol{C}_a) = MVN(\boldsymbol{\mu}_2, \boldsymbol{\Sigma}_2)$$
$$p(\sigma^2|\boldsymbol{C}_e, \boldsymbol{C}_a) = Inv - \chi^2(\nu_2, s_2^2)$$

  - $\mu_1 = f(X_1, C_a), \quad \Sigma_1 = f(X_1, \sigma^2), \quad s_1^2 = f(C_a, X_1, \mu_1, n_1, d), \quad \nu_1 = n_1 - d$
  - $\mu_2 = f(X_2, C_e, \mu_1, X_1), \Sigma_2 = f(X_2, \sigma^2), \quad s_2^2 = f(C_e, X_2, \mu_2, n_2, d, \mu_1, X_1), \nu_1 = n_2 + \nu_1$

Proposed Method – Failure probability from predictive distribution of the fused model

- The fragility conditional on $\boldsymbol{\beta}, \sigma^2$: definition

$$F_v|\boldsymbol{\beta}, \sigma^2 = P(D_v \geq C(\boldsymbol{x})) = \Phi\left(\frac{D_v - \boldsymbol{\beta}'\boldsymbol{x}}{\sigma}\right)$$

- The fragility at a given wind load: obtained point by point for each v by MCMC simulation of (second-stage) posteriors & conditional fragility

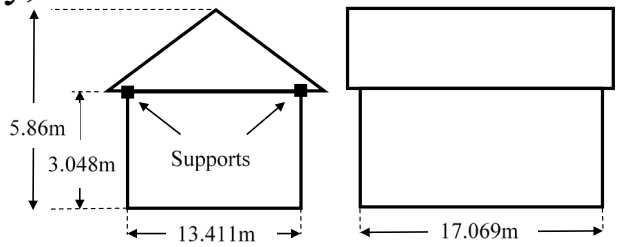
$$F_v = \int \Phi\left(\frac{D_v - \boldsymbol{\beta}'\boldsymbol{x}}{\sigma}\right) p(\boldsymbol{\beta}|\sigma^2, \boldsymbol{C_e}, \boldsymbol{C_a}) p(\sigma^2|\boldsymbol{C_e}, \boldsymbol{C_a}) d\boldsymbol{\beta} d\sigma^2.$$

- Probability of failure: obtained by importance sampling

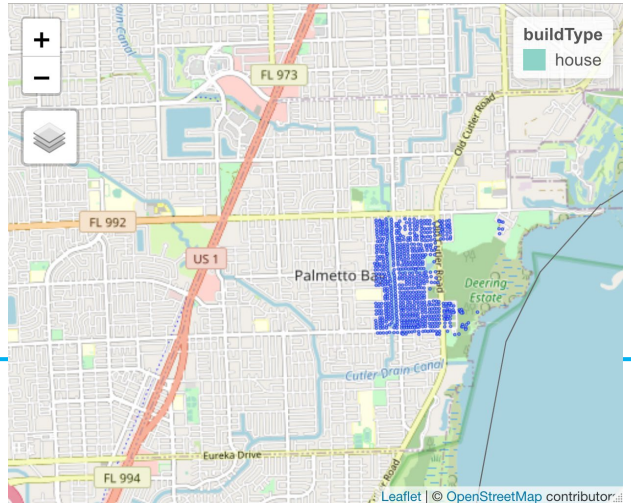
$$P_f = \int_0^\infty F_v p(v) dv$$

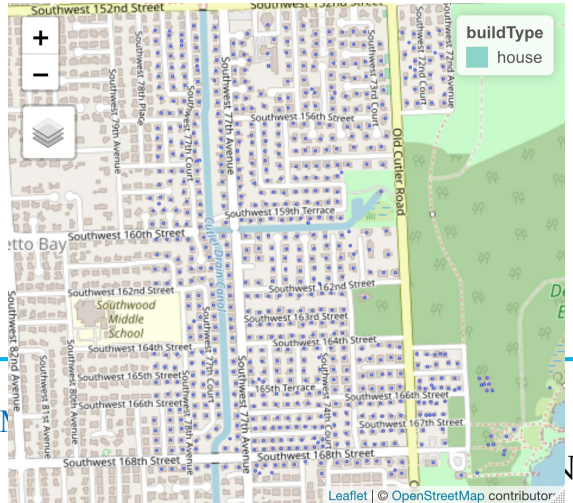
Case study – Miami Dade County Residential Building Inventory Wind Loss Estimation

- Wind speed follows a Weibull distribution
- A gable roof, one story, wood frame residential building



- Building stock subject to the wind loading is obtained from digitized building footprints in OpenStreetMap: inventory of $n = 100$ buildings

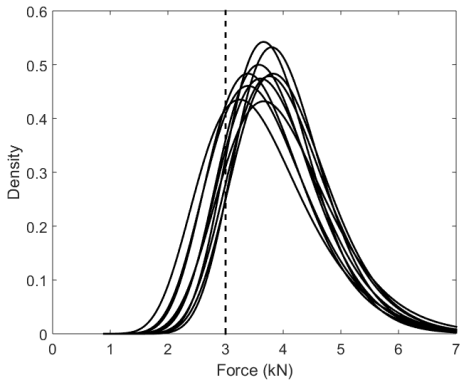


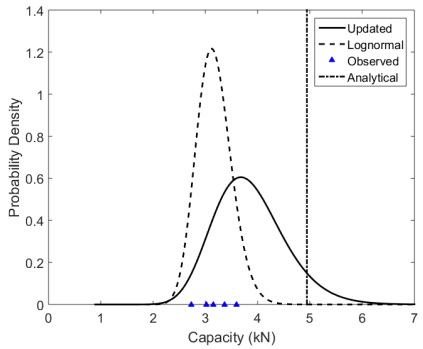


Case study – Miami Dade County Residential Building Inventory Wind Loss Estimation

- Predictive distribution of capacity $\int p(C(\boldsymbol{x})|\boldsymbol{\beta},\sigma^2)p(\boldsymbol{\beta}|\sigma^2,\boldsymbol{C_e},\boldsymbol{C_a})p(\sigma^2|\boldsymbol{C_e},\boldsymbol{C_a})d\boldsymbol{\beta}d\sigma^2$.
  - Lognormal: purely statistical approach (likelihood)
  - Bayesian (Updated): considers both data and the analytical model; for different β and σ^2 sampled from posteriors (left); average of all realizations, a t distribution (right),

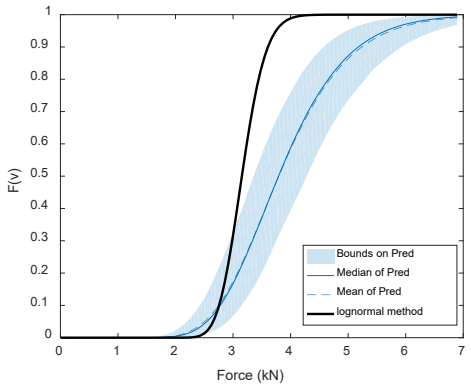
SYP wood
& 2 clips





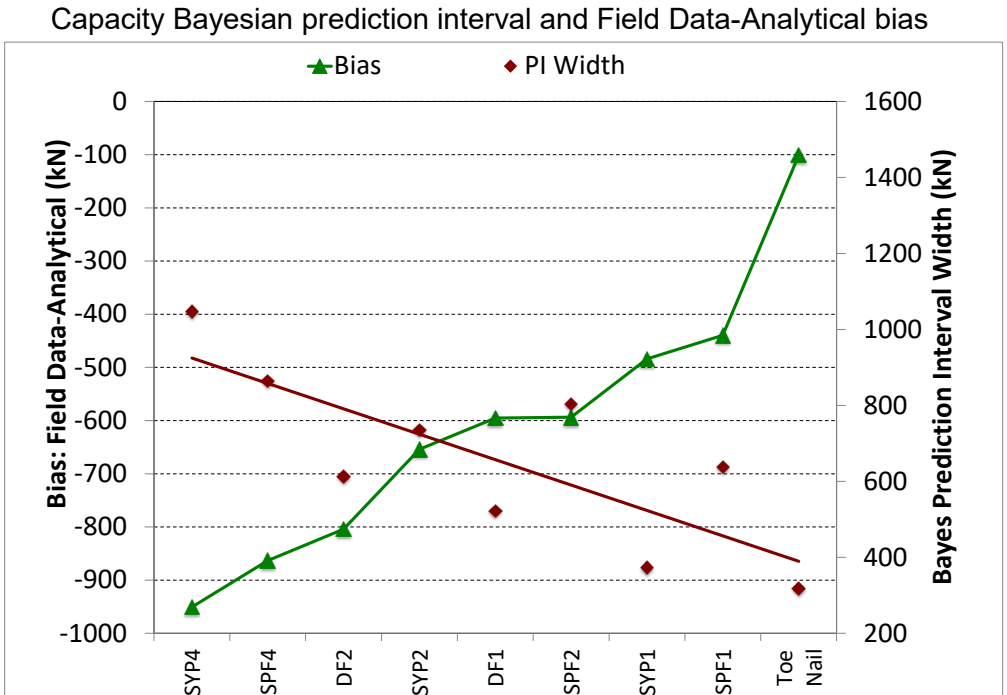
- Determine fragility confidence intervals: Use β and σ^2 sampled from posteriors
  - For each v, sample β and σ^2, find $F_v = \Phi\left(\frac{D_v - \boldsymbol{\beta}'\boldsymbol{x}}{\sigma}\right)$, report 2.5th and 97.5th percentiles

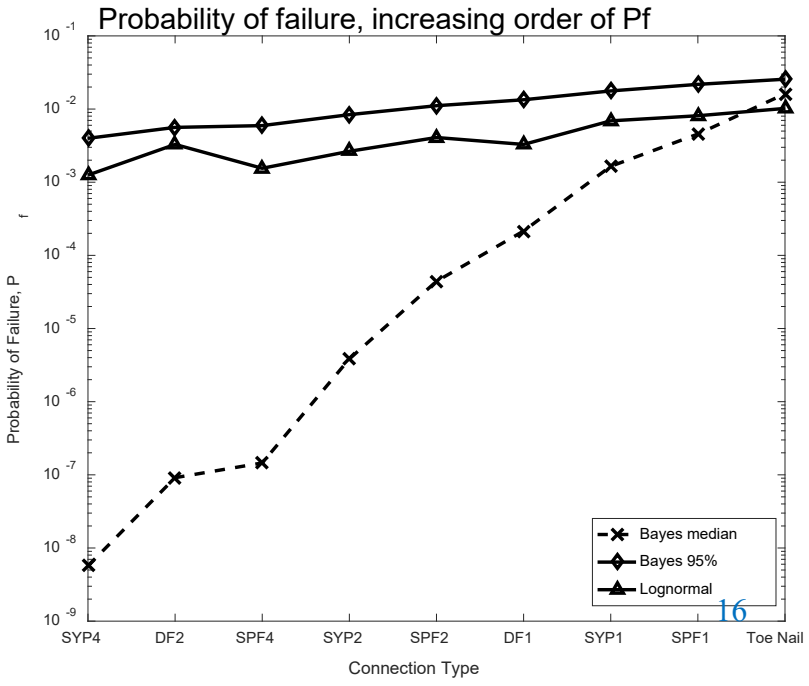
SYP wood
& 2 clips



Case study – Miami Dade County Residential Building Inventory Wind Loss Estimation

- Probability of failure of RTW connections
  - Sample v from wind speed PDF, find $P_f = \frac{1}{m}\sum_i^m F_{v_i}$, repeat: 50th and 95th percentiles of P_f (Bayesian approach) ; mean P_f (lognormal approach)
- Larger bias between analytical prediction & field data → Wider Bayesian Prediction Interval
  - Bayesian median is less conservative than lognormal for all cases, but the 95th percentiles are more conservative





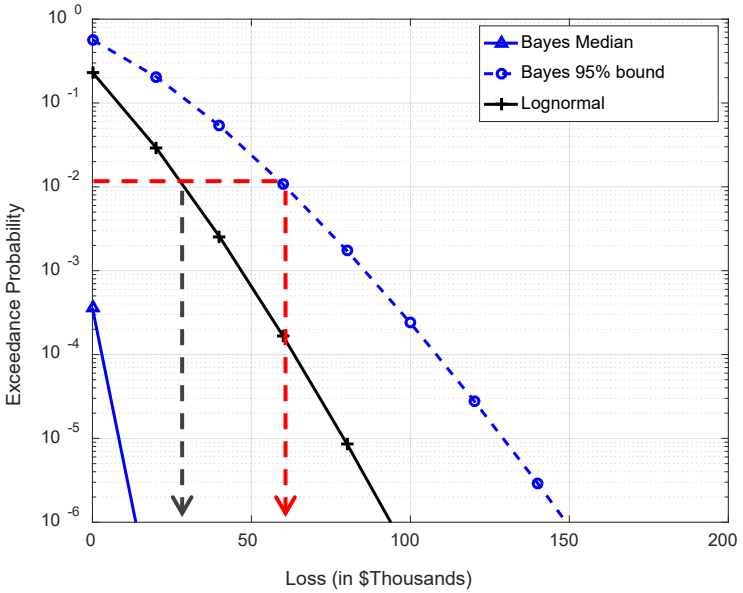
Case study – Miami Dade County Residential Building Inventory Wind Loss Estimation

- From the posterior of P_f, find the posterior of exceedance probability

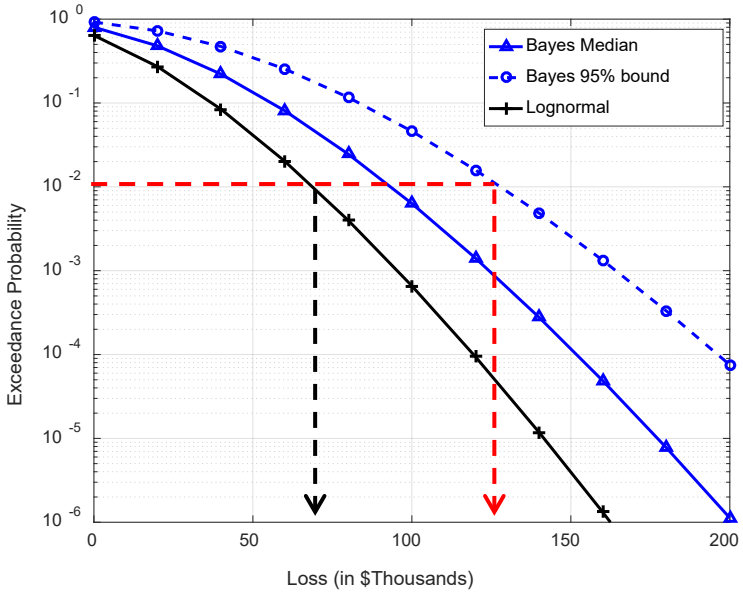
$$EP_l = P(L > l) = P(w > l/L_0) = \sum_{w=\lfloor l/L_0 \rfloor}^{M} p_{RTW}(w)$$

- Median and 95-th percentile of exceedance probability
  - Consider a 100-year hurricane (probability of exceedance 10^{-2})
  - Clips result in aggregate loss upper bound \$61K (Bayesian) (median \$15K) and \$29K w/ lognormal
  - Toenail result in aggregate loss upper bound \$128K (Bayesian) (median \$88K) and \$69K w/ lognormal

SYP wood and 2 clips



Toe nails



Conclusions & Future Research

- Bayesian approach provides a systematic approach to account for large variations in building loss estimates, makes analyzing benefit of mitigation difficult with existing component-based modeling methods
  - Regional loss uncertainty margin ~$45K
  - Benefits of clips over toe-nails: $69K to $29K, w/ lognormal ($128K to $61K, w/ Bayesian)
- Extend uncertainty quantification to the wind speed model parameters
- Account for spatial uncertainty (of building stock and wind speed) in regional loss estimation (all sites do not have equal probability of failure)
- Utility theory to design optimal incentives depending on homeowner risk preferences and hurricane loss risk

Acknowledgement

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Thank you!

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