CRE Part 1: Reliability Fundamentals

Part 1A: Leadership Foundations

RELIABILITY: the probability that an item will perform a required function without failure under stated conditions for a specified period of time.

1. Benefits of Reliability Engineering

Why Reliability Engineering is important:

- Customer expectations
- As products become more complex, the reliability requirements of individual components increase.
- HSE hazards
- Marketing and Warranty
- Competition
- Contractual obligations

The study of reliability engineering responds to each of these influences by helping designers determine and increase the useful lifetime.

2. Interrelationships of Safety, Quality & Reliability

Quality	Reliability
Parts inspected after manufacturing	Parts monitored throughout lifetime
Uses Manufacturing data	Uses operational/failure data
Suggest changes that permit the item to be produced within tolerance at a reasonable cost	Make recommendations that permit the item to function correctly for a longer period of time

3. Reliability Engineer Leadership Responsibilities

A Reliability Leader's primary job is to:

- Instill a vision of "giving the most accurate answer with the data available"
- Influence behavior of others to achieve reliability goals
- Understand the business ramifications of reliability
- Explain reliability and safety risks to management for informed decision making
- Remind team of ethical responsibilities
- Support the decision of the reliability position to management

Key **FACET**s of Leadership:
Focus, **A**uthenticity, **C**ourage, **E**mpathy, **T**iming

4. Reliability Engineer Role & Responsibilities in the Product Lifecycle

Producing a safe product must be a top priority.
The reliability engineer is responsible for/to:

1. Collecting/analyzing failure data
2. Presenting data in an understandable format
3. Ensure key decision makers understand the analyses
4. Consider HSE risks
5. Consider safety/liability hazards even without a failure
6. Could misuse or disposing of the product cause safety/liability hazards
7. Can the malfunction of other parts cause hazards
8. What is the impact of government regulation
9. Does product design compromise the reliability of the components
10. Are the suppliers reliable

Issues such as these must be kept before management.

5. Function of Reliability in Engineering

Reliability techniques are used to:

- Improve product design
- Support warranty documents for marketing and advertising
- Determine when to used warnings/alarms for HSE issues
- Improve manufacturing processes
- Determine suitable maintenance strategies
- Compare vendors

Every function of an organization can benefit.

6. Ethics in Reliability Engineering

ASQ Code of Ethics provides guidelines.

- The reliability engineer's responsibilities are not limited to crunching numbers and producing good analyses but include the promotion of product reliability and safety.
- Efforts should be expended on objective analysis of facts and not on self-promotion.
- When issuing statements, a disclaimer separating one's views from those of the employer is required.
- Divulge any conflicts of interest which may bias conclusions.
- The reliability engineer is required to present both good and bad news scenarios when making recommendations.
- Do not disclose business or technical information without consent
- Ensure the correct person is credited for work done

7. Supplier Reliability Assessments

3 scenarios:

1. Customer assumes all responsibility for reliability engineering and requires supplier compliance (common when customer has full design responsibilities)
2. Supplier assumes all responsibility and may share financial costs (common when supplier has design control)
3. Shared responsibility (common when supply chain is geographically long)

Contracts must be clear on the agreements.
Customer will conduct assessments such as audits or product testing and evaluation.

8. Performance Monitoring

Periodically measuring a project's progress toward reliability, maintainability and safety (RMS) objectives & providing feedback on the results to decision makers to improve performance.

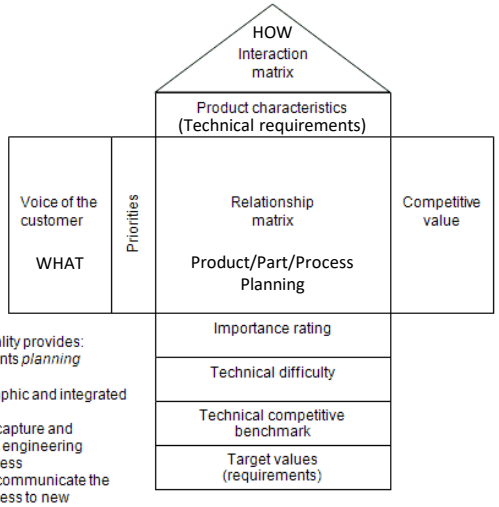
Uses of Performance Indicators:

- Strategic planning
- Performance accounting (deciding resource allocations)
- Forecasting and early warning during program implementation
- Measuring program results
- Program marketing and public relations
- Benchmarking
- Quality management

Performance indicators can flow from customer requirements via a Quality Function Deployment (QFD).

Quality Function Deployment (QFD): a structured approach to defining customer needs or requirements and translating them into specific plans to produce products to meet those needs. The input is the VOC.

Figure 1 — House of quality template and benefits



Throughout a typical product development cycle, tracking parameters in the form of reliability tools and methods are used for performance monitoring (e.g., MTBF, reject rates, number of defects, etc.).

Part 1B: Reliability Foundations

1. Basic Reliability Terminology

Mean life/expected time to failure: the average time to failure of identical products operating under identical conditions.

Mean Time To Failure (MTTF) for non-repairable items.

Mean Time Between Failures (MTBF) for repairable items.

MTTF/MTBF assume exponential failure distribution and constant failure rate.

$$MTTF = \theta = \frac{\Sigma t_i}{n}$$

t = failure times
n = number of items

$$MTBF = \frac{nm}{r}$$

n = number of items
m = number of hours in test
r = number of failures

There are 4 types of failure data:

1. *Exact failure times* – exact failure time is known
2. *Right-censored data* – the failure happened/would have happened after a particular time (e.g. test was stopped when part still functioning)
3. *Left-censored data* – the failure happened before a particular time (e.g. test not monitored continuously)
4. *Interval-censored data* – the failure happened between 2 times (e.g. test is monitored every 5hrs)

Failure rate/hazard function (λ): the number of failures per unit time.

Mean Time to Failure (MTTF): the average time elapsed until the product is no longer performing its function. If the item is repairable, *mean time between failures* is used.

Failure Rate (λ) = 1 / Mean Life (MTBF or MTTF)

Availability is the probability that a product is operable when needed:

$$A = \frac{total\ time\ a\ functional\ unit\ is\ capable\ of\ being\ used}{the\ length\ of\ the\ interval}$$

$$Availability = \frac{average\ uptime}{average\ uptime + average\ downtime}$$

Mean Time To Repair (MTTR): average time taken to return the product to operational status.

Dependability is the probability that a product will function at a particular point in time during a mission

Maintainability is the probability that a failed product will be repaired within a given amount of time once it has failed.

Reliability R(t) is defined as,

$$R(t) = \frac{\#\ units\ functioning\ at\ the\ end\ of\ the\ time\ period}{\#\ units\ that\ were\ functioning\ at\ the\ start\ of\ the\ test}$$

B(X) life: the amount of time that has elapsed when X% of the population has failed.

Mean Time Between Repairs (MTBR): includes info about the type of repair and resources required.

CRE Part 1: Reliability Fundamentals

Mean time between unplanning maintenance action (MTBUMA): indicates the level of confidence that can be placed in the machine when it is placed in a vital role.
Service interval: the recommended time between routine checks and replacements.
Relationship to Product Specifications: consider providing reliability requirements to suppliers rather than specific product characteristics.

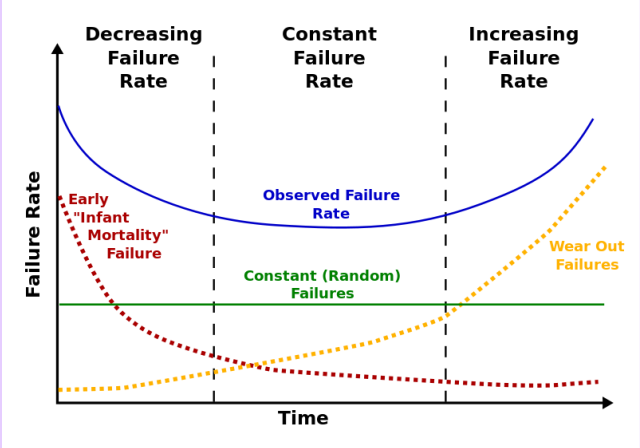
Maintainability

Maintainability: a probability that a particular maintenance activity can be accomplished in a stated period of time. Includes serviceability and repairability.
Serviceability: the level of difficulty encountered when performing the maintenance activity.
Repairability: the level of difficulty when returning a failed item to usefulness.
The ultimate goal is to develop a RCM (reliability centered maintenance) program:

1. Develop a functional hierarchy of the machines and their components
2. Make a complete list of failure modes for the machines
3. Perform an evaluation of each preventive maintenance activity to determine whether it really aids in preventing failure modes.

Product Lifecycle

The bathtub curve shows the 3 stages in the lifecycle of a product.



2. Drivers of Reliability Requirements & Targets

1. Established reliability goals & requirements
2. Product design requirements (from customer needs)
3. Process design
4. Validation & Verification
5. Post-production evaluation
6. Training and education; management support

Design requirements originate from customer needs.
Faster, better, cheaper – pick any two!

3. Corrective and Preventive Action

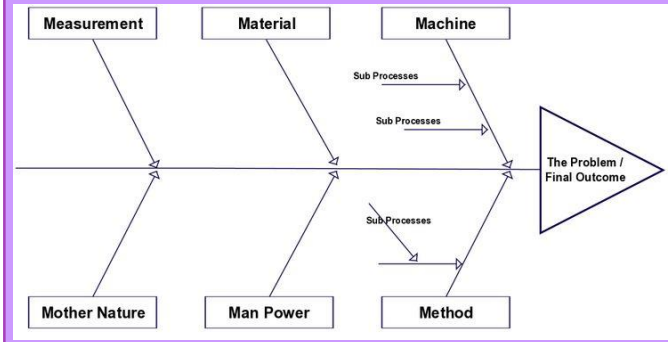
The ultimate test of the effectiveness of a preventive or corrective action is the ability to turn the action on or off and observe the corresponding effect on the process. Data should be collected before and after the installation of the preventive/corrective action. There are tools for determining whether the data provide evidence that the action has been successful:
Histograms: can be used to plot e.g., mean lifetime before/after action. The centers should shift on the graph.
A major part of preventive and corrective action activities is the installation of a system that monitors the process to ensure that the problem doesn't reoccur.
Documentation regarding problem-solving activity should be maintained and available to team members.

4. Root Causes Analysis

Once defects or failures are identified, the root cause(s) must be determined.

Cause-and-Effect Diagram

The fundamental tool to use is the **cause-and-effect**/fishbone/Ishikawa diagram. This diagram divides possible causes into broad categories that help stimulate inquiry as successive steps delve deeper. E.g.:



The fishbone diagram is an example of divergent thinking in that it seeks to go off in all directions, with the aim of finding all possible causes of failure. After completing it, the next step is to perform a convergence exercise to prioritize the failure prevention activities. Various voting schemes can be used, such as a **cause-and-effect matrix,** which relates various causes to customer requirements.
The **five why's** can also be used to dig into the root causes by repeating the question "Why does this happen?".
A **flowchart** of the process can aid in identifying root causes.

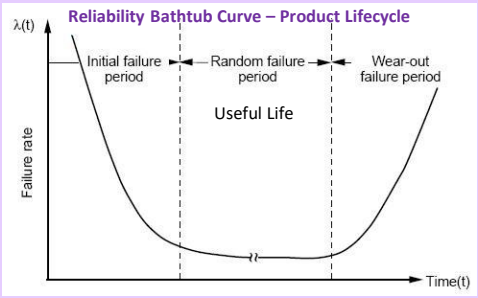
Scatter Diagrams

When several causes for a problem have been proposed, measurements are taken for various levels of the variables suspected of being a cause. Then each variable is plotted against the measured value of the problem to get a rough idea of ***correlation*** or association.

CRE Part 1: Reliability Fundamentals

5. Product Lifecycle Engineering Stages

The *life cycle cost* of a product includes the cost to purchase, operate and maintain the product during its useful lifetime. Increased reliability can mean increased manufacturing cost and selling price – but ultimately a lower life cycle cost.



Improve early failure stage by shortening its length;
Improve useful life stage by decreasing constant failure rate;
Improve wear-out stage by delaying its onset.

During the useful life of the product, the product reliability:

$$R(t) = e^{-\lambda t}$$ t = time elapsed; λ = constant failure rate

6. Economics of Product Maintainability & Availability

System availability is determine by the reliability (the probability of the system not failing) and maintainability (the ability to restore the system to service).

There are several ways to measure availability, depending on the types of downtime which are included in the total time. Failure rate is assumed to be constant.

Inherent availability: excludes preventive maintenance and any logistic downtime:

$$A = \frac{MTBF}{MTBF + MTTR}$$

MTTR includes the time for corrective maintenance actions plus the probability of occurrence of the failure.

Achieved availability: includes the downtime due to corrective and preventive maintenance actions. Does not include any delay time in acquiring supplies, etc. The ***mean time between maintenance actions*** (MTBMA) includes both scheduled and unscheduled maintenance actions:

$$MTBMA = \frac{1}{\lambda + \mu}$$

where,
λ = failure rate;
μ = preventive maintenance rate

The ***mean active maintenance time*** (MAMT) includes the average CM time and the average time to perform PM.

Operational availability: includes all downtime. The mean downtime (MDT) includes logistic/waiting/admin time.

$$A = \frac{MTBMA}{MTBMA + MAMT} \; or \; A = \frac{MTBMA}{MTBMA + MDT}$$

7. Cost of Poor Reliability

See: section 1B5 notes on lifecycle cost

Reliability Engineering for Software Products consists of:

- Error prevention
- Fault detection and removal
- Fault containment through redundancy

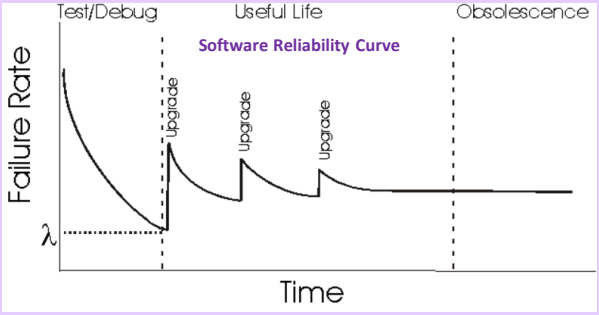
Redundant techniques can be used to automatically detect and contain faults. Such as:

- Use of alternate formulas to calculate a value and compare results
- Use of dual processors and comparing results
- Establishing known limits on a result and flagging values outside the limits

It is better to prevent errors than to detect faults → use thorough **design requirements** prior to code development.

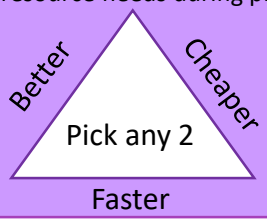
General rules for software code include:

- Make it modular
- Keep it simple = easier to debug and maintain
- Provide documentation and comments



8. Quality Triangle

The Reliability Engineer should work with the Project Manager on time/resource needs during project planning phase.



9. Six Sigma Methodologies

Reliability Engineering tools help design engineers through: MTBF values, failure types, optimal burn-in times, warranty recommendations, impact of age/operating conditions, impact of parallel/redundant design features, accelerated life testing, analysis of field failure data, efficiencies through concurrent engineering, cost accounting estimates through life cycle cost analysis, FMEA/FMECA results.

LEAN: a systematic approach to identifying and eliminating waste (NVA activities) through continuous improvement by flowing the product at the pull of the customer in pursuit of perfection.

NVA: describes a process step/function that is not required for the direct achievement of process output.

Lean Manufacturing eliminates/reduces waste by using:

- Teamwork; Clean, organized workspaces
- Flow systems (not batch-and-queue)
- Pull systems (replenish what customer has consumed)
- Reduced lead times through efficiency improvements

Six Sigma *concepts:*

- Teams working on well-defined projects impacting the bottom-line
- Statistical thinking (Black Belts)
- DMAIC approach to problem solving
- Management support
- Use of tools: statistical process control, control charts, FMEA, process mapping
- 6 sigma = 3.4 defects per million opportunities

LEAN focuses on waste reduction, whereas SIX SIGMA emphasizes variation reduction.
LEAN achieves its goals by using less technical tools, whereas SIX SIGMA uses statistical tools.

10. Systems Engineering and Integration

Systems engineering is a branch of engineering that integrates the activities of other disciplines to produce a product or service. It has 6 steps:

1. Problem statement: investigation of customer needs (different from requirements as the needs may not be understood or articulated by customers)
2. Study alternatives: divergent thinking; feasibility studies. Avoid mission creep – the tendency to add too many new features.
3. Model/Simulate the system: physical/software models to identify problems and opportunities
4. Integrate the various elements: to operate as one
5. Run the system: verify outputs meet requirements
6. Study performance: evaluate the system

These steps may be repeated as loops.

CRE Part 2: Risk Management

Part 2A: Identification

1. Risk Management Techniques

A typical risk management program has 3 key elements:

1. Identification of potentially undesirable effects
2. Risk analysis using techniques such as:
   - FMEA/FMECA
   - Production reliability acceptance test (PRAT)
   - Fault tree analysis (FTA)
   - Success tree analysis (STA)
   - Failure reporting, analysis, and corrective action system (FRACAS)
   - Preliminary hazards analysis (PHA)
   - Failure modes, effects & diagnostic analysis (FMEDA)
   - Systems theoretic process analysis (STPA)
3. Improvement through prevention, intervention, assist and/or rescue,

Consider the **human cognition error**: misunderstanding, misidentification, inexperience, inattention and lack of standards. Principal types of mistake-proofing techniques include:

- Physical barriers to errors
- Visual reminders
- Use of automated equipment
- Standardizing work

Activity must be continuous in the 3 elements of the risk management program.

2. Types of Risk

Scheduling: contingency planning, schedule prioritization
Safety: HSE hazard identification
Financial: exchange rates, customer credit worthiness, etc.
Regulatory: political trends, regulatory rules
Suppliers: financial status, delivery delays
Materials: availability/cost of key materials, supply/demand
Risk is the effect of uncertainty on objectives.
Use a **Risk Assessment Matrix** to list and evaluate the impact of all risks. Reliability data helps to determine occurrence/ probability of failure. identify the vulnerabilities, and implement corrective actions.

Part 2B: Analysis

1. Fault Tree Analysis (FTA)

Fault Tree Analysis can be used to study failure modes. The failure mode being studied is the "top" or "head" event. An FTA considers underlying causes for a failure mode and studies the relationships between various failures. Useful in the early stages of design. Provides a diagram of the complexities to visualize possible preventive/corrective actions. Ask: "what condition(s) could cause this failure?"
Success Tree Analysis approaches a system more positively, asking "What must occur for the system to function successfully?".
AND Gate: output occurs only if **all** inputs occur.
OR Gate: output occurs if **at least one** input occurs.
Voting OR Gate: the output occurs if **k or more** inputs occur, where k is specified, usually on the gate symbol.

Table 1: Classic Fault Tree Gates and their Traditional RBD Equivalents

Name of Gate	Classic FTA Symbol	Description	RBD Equivalent
AND		The output event occurs if all input events occur.	Simple Parallel Configuration
OR		The output event occurs if at least one of the input events occurs.	Series Configuration
Voting OR (k-out-of-n)		The output event occurs if k or more of the input events occur.	k-out-of-n Parallel Configuration
Inhibit		The input event occurs if all input events occur and an additional conditional event occurs.	Simple Parallel Configuration of all the events plus the condition
Priority AND		The output event occurs if all input events occur in a specific sequence.	Standby Configuration: Special case with a switch.
XOR		The output event occurs if exactly one input event occurs.	Cannot be represented. *See discussion in article.*
Sequence Enforcing	Various representations	The output event occurs if all input events occur in a specified sequence.	Cold Standby Configuration
Transfer		Used to indicate a transfer/continuation to another tree.	Subdiagram Block

2. Failure Mode and Effects Analysis (FMEA)

Failure Modes and Effects Analysis (FMEA): an engineering technique for system reliability improvement. The purpose is the anticipation and mitigation of the negative effects of possible failures prior to the time they occur.

Process Step / Input	Potential Failure Mode	Potential Failure Effects	SEVERITY	Potential Causes	OCCURRENCE	Current Controls	DETECTION	RPN
What is the process step and Input under investigation?	In what ways does the Key Input go wrong?	What is the impact on the Key Output Variables (Customer Requirements)?		What causes the Key Input to go wrong?		What are the existing controls and procedures (inspection and test) that prevent either the cause or the Failure Mode?		
								0
								0
								0

1. Gather data
2. List all possible failure modes
3. Calculate a Risk Priority Number (RPN) = SxOxD
4. Develop a corrective action plan & document results

CRE Part 2: Risk Management

Failure Mode, Effects and Criticality Analysis (FMECA): bases the prioritization scheme on probability of occurrence and severity of the failure mode. These can be plotted on a graph to show high severity & frequent failure modes. Additional concerns such as safety, downtime, preventive maintenance, stock of spare parts, can also be included. The emphasis is placed on the CRITICALITY of failure modes.

Severity \ Occurrence	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Other tools for assessing criticality include flowcharts and block diagrams. These graphical tools display the relationships and dependencies of various subsystems and components. Useful for complex systems.

FMEA & FMECA in Design

It is important to perform the FMEA/FMECA in time to economically make changes to the design using the results of the effort.

The FMEA/FMECA is changed to a quantitative analysis by assigning values to the probability of the failure occurring, to the severity of the effect, and the probability that the system controls will detect and eliminate the failure before the design is complete.

Types of FMEA

- Design FMEA: performed on design or product
- Process FMEA: performed on the process
- System FMEA
- Machinery & Equipment FMEA: performed on equipment used in the production system
- Software FMEA

3. Common Mode Failure Analysis

A *common cause failure* occurs when 2 or more components fail at the same time as a result of a shared cause. Recognizing and mitigating common cause failure is a vital part of designing for reliability.

Whether performing Fault Tree Analysis (FTA), hazard analysis, FMEA or a risk matrix, consider if there exist common causes that could result in multiple faults and place special emphasis on eliminating these common causes – since this will often produce the greatest improvement in reliability.

FTA is especially susceptible to missing the total impact of a common cause as it is often performed with narrow focus.

4. Hazard Analysis

A *hazard* is a condition, event or circumstance that could lead to or contribute to an unplanned or undesirable event.

Hazard analysis: a method to assess risk based on the severity and likelihood. 3 fundamental questions are:

1. What might go wrong? The Risk
2. What is the probability that it will go wrong? The Likelihood
3. What are the consequences? The Severity

The hazard is then the combination of the likelihood and severity of the risk. These can be plotted on a risk matrix. Similar to an FMEA, except it does not include detection in the analysis.

5. Risk Matrix

The risk matrix approach is common in reliability engineering and other efforts that involve the identification, classification and prioritization of risk (e.g., FMEA, hazard analysis, FMECA).

List the potential risks and evaluate their severity and likelihood to then determine the risk.

OR, create a matrix listing product/services in the vertical column, with an estimate of the importance of each in another column. Across the top of the matrix a list of risk types can be listed. This matrix provides for multiplying an estimated importance score of an adverse event by an estimate of the probability the event will occur.

Reliability data can improve the risk assessment process by providing more accurate values for probability of failure.

6. System Safety

A typical safety program has 3 key elements:

1. Identification of safety hazards
2. Risk analysis using techniques such as:
    - FMEA/FMECA
    - Production reliability acceptance test (PRAT)
    - Fault tree analysis (FTA)
    - Success tree analysis (STA)
    - Failure reporting, analysis, and corrective action system (FRACAS)
3. Corrective and preventive action

As Low As Reasonable Practicable (ALARP): a principle used in system safety whereby risk is reduced as far as reasonably practicable (before cost becomes grossly disproportionate to the benefit gained).

Again, consider the human cognition errors.

Continuous Effort

Activity must be continuous in the elements of the system safety program. As the higher-priority hazards are corrected/prevented, action can focus on lower-priority items.

Part 2C: Mitigation

Consequences of failure must also be examined. These are typically costs to the customer and ultimately the producer. Reliability acceptance goals will limit failure occurrence and consequent costs. Acceptance criteria will be:

- Functional, Environmental & Time requirements

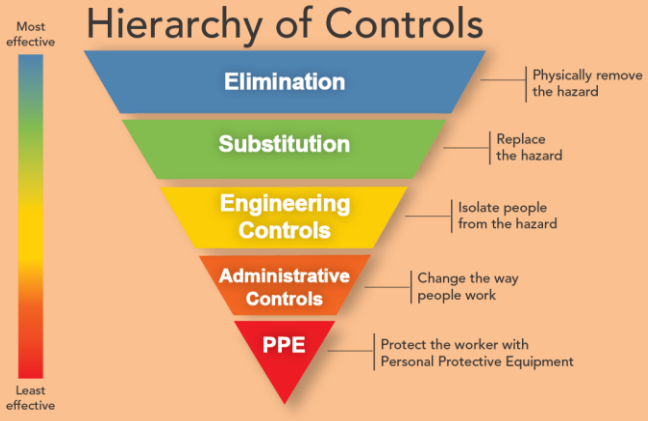
Testing should *validate* that these goals will impact failure rates and consequences.
Reliability goals impose specifications on the product. Testing will *verify* that these specs are being met.

Strategies to Minimize Risk

Once reliability goals have been set and the risks identified, the reliability engineer must consider the risk-benefit analysis and determine the best strategies to minimize the associated risks.

A ***Safety Decision Hierarchy*** can be applied to ensure safety, liability, and regulatory compliance. Use the following activities in order:

1. Eliminate hazards and risks through system design and redesign
2. Reduce risks by substituting less hazardous methods or materials
3. Incorporate safety devices
4. Provide warning systems
5. Apply administrative controls
6. Provide PPE



Product Improvement

This often means redesigning components. The relatively short product lifetimes in electronics and software has caused increased emphasis on obsolescence management.

Obsolescence from the Producer's Viewpoint

When any component of a system is changed, the result is a new system, which must be tested to verify that it performs as specified. Qualifying the component independently from the system may miss undesirable interactions with other components. Judgement is required in defining the level of testing required to qualify the new component. The disposition of obsolete parts must also be managed.

Obsolescence from the Customer's Viewpoint

The warranty documents the responsibility that the supplier has to the customer, e.g.:

a. Commitment to support the product with replacement parts for a stated period of time
b. Lifetime buy (requires careful forecasting)
c. Backwards compatibility of new components with old

CRE Part 3: Probability and Statistics for Reliability

Part 3A: Basic Concepts

1. Basic Statistics

The *probability* that a particular event occurs is a number between 0 and 1 inclusive. *Random* means each event has an equal chance of occurring.

Mean = average; measure of central tendency. For discrete values, this is also called the *expected value* or *expectation*.

Median: middle number when numbers are sorted by size

Mode: the number in the list appearing most frequently

3 measures of **DISPERSION**:

Range: subtract the smallest from the largest value

Standard deviation: quantifies the amount of variation in a dataset. Low σ = values are close to the mean.

Variance: the expectation of the squared deviation of a random variable from its mean:

$$Sample\ Variance = \frac{\sum(x-\bar{x})^2}{n-1}$$

x-bar = mean
n = no of items

$$Sample\ Standard\ Deviation = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

If the entire *population* of data is available,

$$Population\ Standard\ Deviation = \sqrt{\frac{\sum(x-\mu)^2}{n-1}}$$

Where μ = population mean.

Population: the collection of all individuals, items or data under consideration

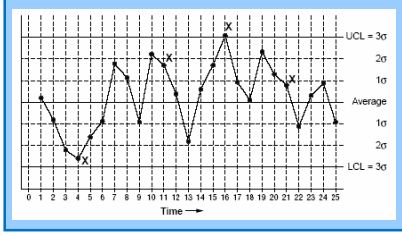
Sample: part of the population from which info is collected

Random sample: each unit has an equal chance of being chosen for the sample

Statistic: a descriptive measure of a sample

The Central Limit Theorem (CLT)

The validity of x-bar control charts comes into question when the population is not normal. The control chart operates under the assumption that the region bounded by +/-3σ contains 99.7% of the points from a stable process. This assumes that the distribution of the points is normal. *The CLT establishes that* when independent random variables are added, their sum **tends toward a normal distribution even if the original variables themselves are not normally distributed**, i.e. the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a finite expected value and variance, will be approx. normally distributed, regardless of the underlying distribution.



Parametric vs. Non-Parametric Statistics

Parametric statistics assume that sample data come from a population that follows a known probability distribution. Non-parametric statistics sample data come from an unknown populations.

Hypotheses Testing

The *hypothesis test* is closely related to confidence intervals. When testing hypotheses, we usually work with small samples from large populations. Because of the uncertainties of dealing with sample statistics, decision errors are possible:

Type I error: occurs when the null hypothesis is rejected when it is in fact true. In most tests of hypotheses, the risk of committing a type I error (known as the alpha risk) is stated in advance along with the sample size to be used.

Type II error: occurs when the null hypothesis is accepted when it is false. The risk associated with this is the beta risk.

The statistical power of a test is another consideration when conducting tests of hypotheses. The power of a test is 1 – β and is expressed as a probability from 0 to 1. the higher the power, the more sensitive the test is in detecting differences where differences exist between the hypothesized parameter and its true value. Increasing the sample size, reducing the variance, increasing the effect size index or decreasing the alpha level will increase the statistical power.

Null Hypothesis, H_0: this is the hypothesis that there is no difference (null) between the population of the sample and the specified population (or between the populations associated with each sample). The null hypothesis can never be proved true, but it can be shown (with specified risks of error) to be untrue, that is, that a difference exists between the populations.

Alternative Hypothesis, H_a: this is a hypothesis that is accepted if the null hypothesis is rejected.

Note: the alternative hypothesis is a statement that contradicts the null hypothesis. The corresponding test statistic is used to decide between the null and alternative hypotheses.

One-Tailed Test: a hypothesis test that involves only one of the tails of a distribution. A one-tailed test is either right- or left-tailed, depending on the direction of the inequality of the alternative hypothesis.

Two-Tailed Test: a hypothesis test that involved two tails of a distribution.

Test Statistic: a statistic calculated using data from a sample. It is used to determine whether the null hypothesis will be rejected.

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Rejection Region: the numerical values of the test statistic for which the null hypothesis will be rejected.
Critical Value(s): the numerical value(s) of the test statistic that determine the rejection region.

Steps in Hypothesis Testing

1. Determine that the conditions or assumptions required for the test are met
2. State the null and alternative hypothesis and determine whether it is a one- or two-tailed test
3. Determine the α value (significance level)
4. Determine the critical values. These are typically found in a table such as the *Z*, *t*, or chi square tables. Use these values to define the reject region.
5. Calculate the test statistic. Each hypothesis test type has a formula for the test statistic. Some of the inputs to the formulas come from the sample data.
6. Determine whether the null hypothesis should be rejected by checking if the value of the test statistic falls in the reject region.
7. State the conclusion in terms of the original problem.

Hypothesis Tests for Means

The hypothesis test usually studied first is the one-sample z-test for population mean. Its steps are:

1. Conditions:
    a. Normal population or large sample (n≥30)
    b. σ known
2. H_0: $\mu=\mu_0$ and H_a: $\mu\neq\mu_0$ or $\mu<\mu_0$ or $\mu>\mu_0$. This is a two-tailed test when H_a has the ≠ sign; left-tailed test when H_a has the < sign; right-tailed test when H_a has the > sign
3. Determine the α value
4. Determine the critical values:
    a. For a two-tailed test, use a z-table to find the value that has an area of α/2 to its right. This value and its negative are the two critical values. The reject region is the area to the right of the positive value and the area to the left of the negative value.
    b. For a left-tailed test, use a z-table to find the value that has an area of α to its right. The negative of this value is the critical value. The reject region is the area to the left of the negative value.
    c. For a right-tailed test, use a z-table to find the value that has an area of α to its right. This value is the critical value. The reject region is the area to the right of the positive value.
5. Calculate the test statistic:

$$z = (\bar{x} - \mu_0)\frac{\sqrt{n}}{\sigma}$$

6. If the test statistic is in the reject region, reject H_0. otherwise, do not reject H_0.
7. State the conclusion in terms of the problem.

In many applications, the population standard deviation is not known. As with the confidence interval, the appropriate distribution is found in the t-table. The below steps change from above:

1. Condition: population is normally distributed or n≥30
4. The critical values are obtained from the t-table using degrees of freedom of n-1
5. The formula for the test statistic is

$$t = (\bar{x} - \mu_0)\frac{\sqrt{n}}{s}$$

Where s is the sample standard deviation. This hypothesis test is referred to as the t-test for one population mean.
See the CRE BOK for:

- Hypothesis tests for means of two populations
- Paired-comparison tests
- Hypothesis test for two population standard deviations
- Goodness-of-fit tests
- Hypothesis tests for proportions

2. Basic Probability Concepts

Complementation Rule:
The probability that an event A will not occur =
1 – (the probability that A does occur) = **P(not A) = 1 – P(A)**
Special Addition Rule:
P(A or B) = P(A) + P(B), if A & B can't occur simultaneously.
The General Addition Rule:
To accommodate events which can happen simultaneously,
P(A or B) = P(A) + P(B) – P(A&B)
Mutually exclusive/disjoint: 2 events which cannot occur at the same time.
Contingency Table: a table showing the distribution of one variable in rows and another in columns, used to study the association between the two variables/calculate probabilities using the above formulas. E.g.:

	Dog	Cat	Total
Male	42	10	52
Female	9	39	48
Total	51	49	100

Conditional Probability: the probability of an event B, given that another A has already occurred:

$$P(B|A) = \frac{P(A\&B)}{P(A)}$$

***General Multiplication Rule*:**
P(A&B) = P(A) x P(B|A)

CRE Part 3: Probability and Statistics for Reliability

Independence and the Special Multiplication Rule

2 events are independent if the occurrence of one does not affect the probability that the other occurs:

P(B|A) = P(B)

Substituting this into the general multiplication rule:

P(A&B) = P(A) x P(B) – only if A and B are independent.

And, **P(A or B) = P(A) + P(B) – P(A).P(B)**

Multiple Events:

$$Probability = P = \frac{no.of\ ways\ an\ event\ can\ occur}{no.of\ possible\ outcomes}$$

To calculate number of possible outcomes, there are 2 ways:

Combinations:

Number of combinations of *r* objects (e.g. defects) from a collection of *n* objects =

$$C = \frac{n!}{r!\,(n-r)!}$$

Where ! means factorial: 6! = 6x5x4x3x2x1

Note: Combinations are used when order is not significant.

Permutations:

Used when the order does matter.

Number of permutations of *r* objects (e.g. defects) from a collection of *n* objects =

$$C = \frac{n!}{(n-r)!}$$

3. Probability Distributions

Discrete distribution: distributions based on random variables that can take on only (countable) integer values, or isolated and distinct values.

Continuous distribution: distributions based on random variables that can take on an infinite number of values in a finite interval.

4. Probability Functions

DISCRETE DISTRIBUTIONS:

Binomial distribution: a discrete distribution whose random variable can take on one of only two possible values.

Binomial Formula:

$$P(X = x) = \frac{n!}{(n-x)!\,x!}p^x(1-p)^{n-x}$$

Where,

n = sample size; x = number of failures;

p = proportion of the population that has failed

P(X=x) = the probability that the sample has x failures

The binomial distribution consists of the set of possible x-values and their associated probabilities.

The *probability density function* (PDF) is the expression that generates the distribution (the binomial formula).

The *cumulative distribution function* (CDF) is the sum of the probabilities up to and including the x-value:

$$F(x) = P(X \le x) = \sum_{t \le x} P(X = t)$$

The mean and standard deviation of a binomial distribution are given by:

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}$$

Poisson distribution: a discrete probability distribution that may be used to find the probability that an event will occur a specified number of times:

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

x = a whole number

λ = a real number

The *cumulative density function* for the Poisson distribution is:

$$\sum_{t \le x} P(X = t) = \sum_{t \le x} e^{-\lambda}\frac{\lambda^t}{t!}$$

The mean and standard deviation of the Poisson distribution are given by:

$$\mu = \lambda \qquad \sigma = \sqrt{\lambda}$$

CONTINUOUS DISTRIBUTIONS:

Exponential distribution: frequently used to model time to failure for products when the failure rate is constant.

The PDF is:

$$f(t) = \lambda e^{-\lambda t}$$

Where, λ = constant failure rate, t = time

The CDF for the exponential distribution is:

$$P(X \le a) = F(a) = \int_0^a \lambda\, e^{-\lambda t} dt = 1 - e^{-\lambda t}$$

The CDF can be used to determine the probability of failure during the first t hours.

The probability that a unit is still operating a t hours is:

P(operating at time t) = (1 – probability it has failed by time t) = $e^{-\lambda t}$

P(operating at time t) is the reliability at time t, or R(t).

By definition, the MTTF is 1/λ. Therefore, the reliability at MTTF is,

$$R(MTTF) = R\left(\frac{1}{\lambda}\right) = e^{-\lambda x 1/\lambda} = e^{-1} \approx 0.368 \approx 37\%$$

This shows that at MTTF only about 37% of the products are operating.

Weibull distribution: the PDF for the Weibull distribution is,

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

Where,

t ≥ 0, shape parameter β ≥ 0, scale parameter ≥ 0.

Various shapes are possible by selecting different values of β. If β = 1, the Weibull reduces to the exponential, and if β ≈ 3.44, the curve approximates the normal distribution.

The hazard function is,

$$h(t) = \frac{\beta}{\eta^{\beta}}(t)^{\beta-1}$$

The CDF for the Weibull is,

$$\mathrm{F}(t) = 1 - e^{-\left(\frac{1}{\eta}\right)^{\beta}}$$

Again,

$$\mathrm{R}(t) = 1 - \mathrm{F}(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

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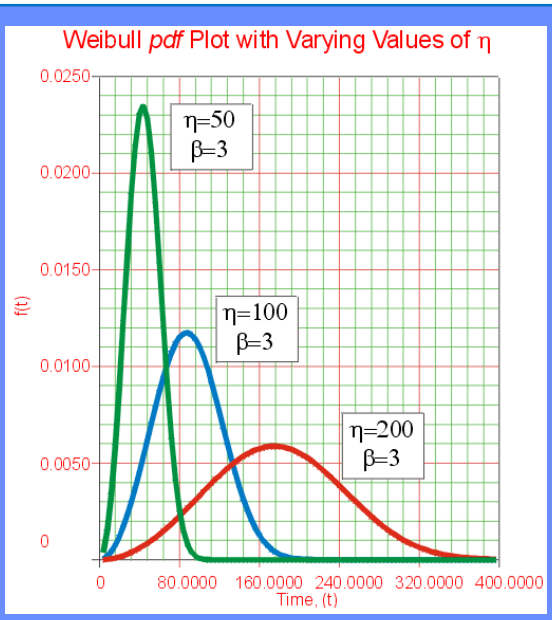


Normal distribution: considered the most important distribution. The PDF is,

$$f(x) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

Where,
μ = mean, and σ = standard deviation. Changes in the mean cause the center of the distribution to be moved left or right along the x-axis. As standard deviation increases, the distribution becomes narrower, centered around the mean. The standard normal table can be used to find specific areas under a standard or non-standard normal curve if the mean and standard deviation are known.

Lognormal distribution: if the natural logarithm (ln) of a random variable is normally distributed, the variable follows the lognormal distribution. The PDF is,

$$f(x) = \frac{e^{-0.5\left(\frac{x'-\mu_{x'}}{\sigma_{x'}}\right)^2}}{x\sigma_{x'}\sqrt{2\pi}}$$

Where,
x' = lnx, $\mu_{x'}$ = mean of the x' values, $\sigma_{x'}$ = standard deviation of the x' values
The lognormal distribution has been found to be a good mathematical model for times to failure for some electronic and mechanical products (transistors, bearings, electrical insulation). The mean and standard deviation are given by,

$$\mu = e^{\mu_{x'}+0.5\sigma_{x'}^2}$$

$$\sigma = \sqrt{\left(e^{2\mu_x+\sigma_{x'}^2}\right)\left(e^{\sigma_{x'}^2}-1\right)}$$

Descriptive Characteristics of Distributions

Skewness of a distribution is defined as,

$$Skewness = \frac{n}{(n-1)(n-2)}\sum\left(\frac{x_j-\bar{x}}{s}\right)^3$$

Where,
n = sample size,
s = sample standard deviation,
x_j= sample values
If skewness is 0, the distribution is symmetric about its mode. If skewness < 0, the distribution is left-skewed; if skewness > 0, the distribution is right-skewed.

Kurtosis is a measure of flatness and is defined as,

$$kurtosis = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left(\frac{x_j-\bar{x}}{s}\right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$

A normal distribution has kurtosis = 0. If kurtosis < 0, the distribution is flatter than the normal distribution. If kurtosis > 0, the distribution is more peaked than normal.

5. Sampling Plans for Statistics & Reliability Testing

If the width of the confidence interval is specified in advance, it is then necessary to calculate the sample size that will meet that specification. The table below shows formulas for this task.
E denotes the margin of error, which is half the width of the confidence interval.
The confidence level is 1 – α.

Confidence Interval for	Sample size n Round to nearest whole #
μ, population mean	$\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$
p, population proportion	$\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 \hat{p}(1-\hat{p})$ $\hat{p}$ denotes an estimate for p
p, population proportion	$0.25\left(\frac{z_{\alpha/2}}{E}\right)^2$ Use when no estimate for p is available
$\mu 1 - \mu 2$, the difference between 2 population means	$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2(\sigma_1^2+\sigma_2^2)}{E^2}$
$p1 - p2$, the different between 2 population proportions	$n_1 = n_2 = 0.5\left(\frac{z_{\alpha/2}}{E}\right)^2$

Rachel Stanford 2018
stanford.rachel@gmail.com

Reference:
CRE BOK 3rd Edition

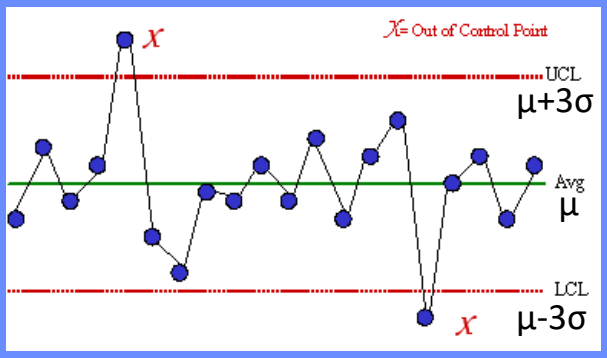
CRE Part 3: Probability and Statistics for Reliability

6. Statistical Process Control (SPC) and Process Capability Studies (Cp, Cpk)

The central tool of statistical process control is the ***control chart***, whose purpose is to provide an early signal when a process changes. During the operation of the process, values are plotted on the control chart.

There are 2 general classifications of control charts: ***variables*** (things that can be measured) and ***attributes*** (things that can be counted) charts.

An in-control process is experiencing *common-cause variation*, while an out-of-control process exhibits *special cause variation*.



Variables control charts:

Type	Distribution	Sample	Application
$\bar{X}$ and R	Normal	2 ≤ 10	Measurement subgroups
$\bar{X}$ and s	Normal	> 10	Measurement subgroups

Attributes control charts:

Type	Distribution	Sample	Application
c	Poisson	Constant	Count # defects/item
u	Poisson	Varies	Count # defects/item
np	Binominal	Constant	Count of defective items
p	Binominal	Varies	Count of defective items
g	Binominal	Individual	Interval between rare events

Variables or Attributes control charts:

Type	Distribution	Sample	Application
X and mR	Normal	1	Individual counts or measurements

The following rules should be used to properly indicate that a process is out of control:

1. 1 point beyond the 3 sigma control limit
2. 8 or more points on one side of the centerline without crossing
3. 4 out of 5 points in the zone B or beyond
4. 6 points or more in a row steadily increasing or decreasing
5. 2 out of 3 points in zone A
6. 14 points in a row alternating up or down
7. Any noticeable/predictable pattern, cycle or trend

*these rules can vary depending on the source.

The target value is closely related to process *accuracy*. The range chart is closely associated with process *precision* (spread or dispersion).

Various process characteristics may be plotted, including physical measurements like dimensions, weights, hardness, or inputs like pressure, voltage.

Variables Charts

Used when measurements on some continuous scale are to be plotted, e.g., length, weight, light intensity, pH, etc. A continuous scale has an infinite number of values between each pair of values. Common variables control charts are the $\bar{X}$ and R chart, ImR (individuals and moving range) chart, and the median chart.

$\bar{X}$ and R chart: values are calculated by averaging the sample of readings taken at a particular time, and the range values are found by subtracting the low value from the high value for each sample.

The standard deviation is messy to calculate, so the standard way to locate control limits is to use formulas using control limit constants:

$$Averages\ Chart: \bar{\bar{x}} \pm A_2\bar{R}$$
$$Ranges\ Chart: LCL = D_3\bar{R}; UCL = D_4\bar{R}$$

Where,

$\bar{\bar{x}}$ = the average of all the average values (the process mean)

$\bar{R}$ = the average of all the range values

A_2, D_3 and D_4 are constants dependent on sample size.

$\bar{X}$ and s chart: uses standard deviation instead of range. Different control limit formulas and constants are used:

$$Averages\ Chart: \bar{\bar{x}} \pm A_3\bar{s}$$
$$Standard\ deviation\ Chart: LCL = B_3\bar{R}; UCL = B_4\bar{R}$$

Where,

$\bar{s}$ = the average of all the sample standard deviation values

CRE Part 3: Probability and Statistics for Reliability

Attribute Charts

Used for count data (e.g., good/bad). The *p*- and *np*- charts are used for plotting **nonconforming items**. If the samples to be tested are of the same size, the np-chart may be used. If the sample size varies, the p-chart should be used.

If each item may have several flaws, nonconformities are counted. The *c*- and *u*- charts are used for plotting **nonconformities**. If the samples to be inspected are of the same size, the c-chart may be used. If the sample size varies, the u-chart should be used.

The p-chart is the most widely used attribute control chart.

The control limit formula for the **p-chart** is:

$$\bar{p} \pm \sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

where,
$\bar{p}$ = average value of p
$\bar{n}$ = average sample size

The control limits for the **np-chart** are:

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$

where,
$\bar{p}$ = average number of defectives per sample
n = sample size

The control limit formula for the **u-chart** is:

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

where,
$\bar{u}$ = average defect fraction =

$$\bar{u} = \frac{\Sigma\, number\ of\ defects}{\Sigma\, sample\ sizes}$$

$\bar{n}$ = average sample size

The control limit formula for the **c-chart** is:

$$\bar{c} \pm 3\sqrt{\bar{c}}$$

where,
$\bar{c}$ = average number of defects

Individuals and Moving Range (ImR) chart: used when the sample size is 1. the moving range is the absolute value of the difference between the current and previous readings. This means the first reading will have no moving range.

The control limit formulas for the ImR chart are:

$$Individuals\colon\ \bar{x} \pm E_2\bar{R}$$

$$Moving\ Range\colon LCL = D_3\bar{R}; UCL = D_4\bar{R}$$

The sample size is the width of the moving window. The moving range points are *correlated* – this means that the only points on the MR chart that signal a process change are those outside the control limits.

Process Capability

Capability analysis: the use of data from control charts.

The 2 principal *capability indices* are:

$$C_p = \frac{USL - LSL}{6\sigma}$$

Where,
USL and LSL are upper and lower specification limits,
σ = process standard deviation

$$C_{pk} = \frac{min[Z_U, Z_L]}{3}$$

Where,

$$Z_U = \frac{USL - \bar{\bar{x}}}{\sigma} \qquad Z_L = \frac{\bar{\bar{x}} - LSL}{\sigma}$$

Where $\bar{\bar{x}}$ = process mean.

In general, C_p shows how good C_{pk} could be if the process were centered.

Historically, a process was considered capable if both C_p and C_{pk} were > 1. Note that since the formula for C_p doesn't use the process mean, it doesn't detect a non-centered process.

7. Confidence and Tolerance Intervals

Point estimation involves the use of sample data to calculate a single value (a statistic) which is to serve as a "best guess" or "best estimate" of an unknown (fixed or random) population parameter.

An estimator is called *unbiased* if the mean of all possible values is equal to the parameter being estimated.

One estimator for a parameter is called more *efficient* than another if it requires fewer samples to obtain an equally good approximation. If 2 estimators A and B are unbiased, A is defined to be more efficient than B if it has a smaller variance. The efficiency E of A relative to B is,

$$E = \frac{\sigma_A^2}{\sigma_B^2}$$

Maximum Likelihood Estimates

Maximum likelihood estimation (MLE) is a method of estimating the parameters of a statistical model given observations, by finding the parameter values that maximize the likelihood of making the observations given the parameters. The likelihood function is given by:

$$L(Y) = f(x_1, Y)\, f(x_2, Y) \ldots f(x_n, Y)$$

In order to find the value of Y that will maximize L(Y), set its derivative equal to zero and solve the resulting equation for Y, i.e., solve L'(Y) = 0. This will produce the value of Y that maximizes the probability that the randomly selected numbers will have values x_1, x_2, …, x_n.

Point Estimates: a point estimate of a population parameter can be made using the values of a sample from that population. One population parameter often estimated is the mean.

Consistency: it is important for estimators to be consistent. Consider a series of estimators for some parameter E and denote the elements of the series E_1, E_2, …, E_n. The estimator is called consistent if E_n approaches E as *n* gets larger.

The Estimate of the Mean: the population mean μ can be estimated using the sample mean $\bar{x}$. If the sample consists of n values, x_1, x_2, …, x_n, the estimate of population mean can be calculated as,

$$\hat{\mu} = \bar{x} = \frac{\sum x(i)}{n}$$

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The Estimate of the Standard Deviation: the population standard deviation σ can be estimated using the sample standard deviation, s.

$$\hat{\sigma} = s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

The point estimates of the population mean and standard deviation are distribution free. This means they are valid regardless of the population distribution.

Confidence Intervals

A statistical confidence interval incorporating a given risk α can be placed about the point estimate of any population parameter made from a sample taken from the population. The statement can then be made that the true parameter of the population is within the interval with a confidence of 1 – α, α = the *significance/risk parameter isn't in the interval*

The Normal Distribution

If the population distribution is normal and the standard deviation is **known**, the confidence interval for the mean is,

$$\bar{x} \pm \frac{Z_{\alpha/2}}{\sqrt{n}}\sigma$$

where,

α = probability that the population mean is not in the interval

1 – α = the confidence level

$Z_{\alpha/2}$ = the value from the Z-table (area under the normal curve) with an area of α/2 in the tail of the distribution.

Minimal error will be incurred if the sample size is >30.

When a confidence interval for the mean of a population is calculated, the margin of error is given by,

$$E = \frac{Z_{\alpha/2}}{\sqrt{n}}\sigma$$

To determine the sample size necessary to obtain a given margin of error, solve for n rounded to a whole number:

$$n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2$$

If the standard deviation of the population is **not known** and must be estimated from the sample data, the *t*-distribution is used to set the confidence limits. The confidence interval for the mean becomes:

$$\bar{x} \pm \frac{t_{\frac{\alpha}{2},v}}{\sqrt{n}}s$$

where,

s = the estimate of the population standard deviation

$t_{\frac{\alpha}{2},v}$ is the value from the t-table for a risk of α and v = n – 1 degrees of freedom.

Confidence Interval on the Standard Deviation

The chi square distribution is used to calculate confidence limits on the standard deviation of the normal distribution. The chi square distribution is not symmetrical, therefore both the upper and lower values must be found from the tables. The chi square values are also defined using degrees of freedom. The chi square value is dependent on the sample size. The confidence interval for the population standard deviation is given by,

$$\sqrt{\frac{n-1}{\chi^2_{(\frac{\alpha}{2},v=n-1)}}}s < \sigma < \sqrt{\frac{n-1}{\chi^2_{(\frac{1-\alpha}{2},v=n-1)}}}s$$

χ^2 is the value from the chi square table using n – 1 degrees of freedom.

Statistical Tolerance Interval

Used to determine confidence values on the area under the normal curve when using a mean and a standard deviation estimated from a sample. The two-sided tolerance interval is $\bar{X} \pm Ks$, where K is a value from the two-sided tolerance table and is dependent of the sample size, the desired confidence level and the proportion of the area to be included within the interval.

If the one-sided limit is desired, the tolerance value is $\bar{X} - Ks$ where K is a value from the one-sided tolerance table.

Exponential Distribution

The exponential distribution is expressed as:

$$f(t) = \frac{1}{\theta}e^{\frac{-t}{\theta}}$$

The mean is θ = MTTF/MTBF. The exponential distribution is unique in that its failure rate is constant:

$$\lambda = \frac{1}{\theta}$$

The estimate of the mean is given by,

$$\hat{\theta} = \frac{T}{r} = \frac{Total\ time\ on\ all\ units}{number\ of\ failures}$$

Confidence Interval for the Mean of the Exponential Distribution

The chi square distribution is used to calculate the confidence limits on the mean of the exponential distribution.

For the two-sided 1-α confidence interval, the lower limit is given by the below if the test is time censored (ended at a given time):

$$\theta_L = \frac{2T}{\chi^2_{(\frac{\alpha}{2},v=2r+2)}}$$

Or if the test is failure censored (ended on a given failure):

$$\theta_L = \frac{2T}{\chi^2_{(\alpha/2,v=2r)}}$$

The difference in the equations is in the number of degrees of freedom for the χ^2 term (either 2r+2 or 2r).

The upper limit for both time-censored and failure-censored tests is,

$$\theta_U = \frac{2T}{\chi^2_{(1-\alpha/2,v=2r)}}$$

For the one-sided confidence limit, the full α risk is placed in the lower tail:

If the test is time censored:
$$\theta_L = \frac{2T}{\chi^2_{(\alpha,v=2r+2)}}$$

If the test is failure censored:
$$\theta_L = \frac{2T}{\chi^2_{(\alpha,v=2r)}}$$

Binomial Distribution

The binomial is a discrete distribution, i.e., only defined for integer values. It gives the probability of exactly *x* success outcomes out of *n* possible outcomes if the probability *p* of success remains the same for each trial.

The PDF for the binomial is,
$$P(x) = C_n^x p^x (1-p)^{n-x}$$
C_n^x is the number of combinations that can be made from *n* items taken *x* at a time:
$$C_n^x = \frac{n!}{x!\,(n-x)!}$$
When the binomial is used to model reliability data, *x* could be the number of units that fail, and is denoted by *r*. If *x* becomes the number of units that do not fail, it is denoted as *n* – *r*. The probability of success on each trial is *p*, which is the mean of the distribution.

Estimating the Binomial Mean from a Sample

An estimate of *p* can be made using data from a test:
$$\hat{p} = \frac{n-r}{n} = \frac{number\ of\ units\ which\ do\ not\ fail}{number\ of\ units\ tested}$$

Confidence Interval for the Mean of the Binomial Distribution

The F-distribution is used to calculate the confidence limits on the mean of the binomial. The F-value is the ratio of two chi square values and has 2 sets of degrees of freedom.

The lower 1-α confidence limit is given as:
$$R_L = \frac{n-r}{(n-r)+(r+1)F_{\alpha,v1,v2}}$$
$F_{\alpha,v1,v2}$ is a value from the F-distribution table with a risk of α and v_1 degrees of freedom for the numerator and v_2 degrees of freedom for the denominator.
$$v_1 = 2(r+1), \quad v_2 = 2(n-r)$$

Success Testing

If there are no failures during a test, a point estimate of the mean cannot be made. A lower confidence value for the mean can be calculated. The lower 1-α confidence value for a no-failure test is given by,
$$R_L = (\alpha)^{1/n}$$
If the test can be run with no failures, the required sample size can be found by solving the equation for *n* and round up to the next integer:
$$n = \frac{\log(\alpha)}{\log(R_L)}$$

Part 3B: Data Management

1. Sources and Uses of Reliability Data

Warranties affect many functions:
- Product pricing
- Supply chain agreements to cover liabilities/procedures
- Accounting procedures to cover warranty costs
- Operation instruction manuals to mitigate failures
- Engineering/reliability must determine causes and corrective actions

Warranties should include:
- Date & duration of warranty
- Definition of parts/defects covered
- Return, replacement, reimbursement, NFF procedures
- Conditions of use, storage, transport, installation

Warranty Data to Collect:
- Traceability of each component (supplier, date, conditions of manufacture and assembly)
- Conditions of use
- Failure parameters and action taken

This data can improve customer satisfaction through design & mfg/operation/installation processes, product recalls, supplier performance, FMEA processes.

Sources of Reliability Data:

Examples of external reliability data sources:

Government-Industry Data Exchange Program (GIDEP)
Institute of Electrical & Electronics Engineers (IEEE)
Rome Air Development Center (non-elec parts)

Internal sources of reliability data:

Source	Comments	Advantages	Limitations	Potential contribution to product reliability
Prototype	Early in design process	Provides visible and testable item	Spending effort on preliminary design	Can head off early design flaws
Develop-ment	Throughout development process	Can track reliability changes with each design	Time/ resources required	Can head off early design flaws
Final Product	Test as soon as available from mfg.	Early basis for warranty calculations	May not be able to simulate environmental conditions	More extensive testing may flag weaknesses
Field Warranty	Need complete data	Customer viewpoint	Data often sketchy	Helps put dollar sign on reliability

CRE Part 3: Probability and Statistics for Reliability

2. Types of Data

Discrete (attributes) data are obtained when the characteristic being studied can have either a finite number or countably infinite number of possible values. *Attribute control charts* are used to plot discrete data on nonconformities (defects) or nonconforming units (defectives), for example:

Attributes control charts for binomial data:

Values for binomial data are classified into one of two categories such as pass/fail or go/no-go. Binomial data are often used to calculate a proportion or a percentage, such as the percentage of sampled parts that are defective.

Use either the P chart or the NP chart to plot nonconforming units. The main difference between P and NP charts is the vertical scale.

- P charts show the proportion of nonconforming units on the y-axis.
- NP charts show the whole number of nonconforming units on the y-axis.

Attributes control charts for Poisson data:

Values for Poisson data are often counts of defects or events. Poisson data are often used to model an occurrence rate, such as defects per unit.

Use either the U chart or the C chart to plot nonconforming units. The main difference between U and C charts is the vertical scale.

- U charts show the number of nonconformities per single unit on the y-axis.
- C charts show the number of nonconformities per sample, which can include more than 1 unit on the y-axis.

Continuous (variables) data are obtained when the characteristic being studied can have any value in a range of numbers. *Variables control charts* are used to plot continuous data. Examples of *univariate* charts are:

Xbar chart: Plots the process mean over time. Used to track the process level and detect the presence of special causes affecting the mean.

R chart: Plots the process range over time. Used to track process variation and detect unexpected variation.

I chart: Plots individual observations over time. Used to track the process level and detect the presence of special causes.

Moving range chart: Plots the moving range over time. Used to track the process variation and detect the presence of special causes.

Multivariate control charts: show how correlated variables jointly affect a process or outcome.

Remember: data can be exact, left-censored, right-censored, or interval-censored.

3. Data Collection Methods

Data can be subjective or objective.

Data from a lab during early testing can be "clean".

Field service data is information gathered from products in use by customers and is affected by many parameters. However, these parameters represent real-world applications.

Once the sources of data have been determined, it is necessary to form the data collection plan, and confirm:

- Who will collect the information?
- When will the data be collected and in what format?
- What measurement equipment will be employed?
- What measures will be used to verify data accuracy and integrity?

4. Data Summary and Reporting

Data must be communicated well to allow for good decision making. The first steps in summarizing data are to construct a histogram and a run or control chart. These help determine whether the process is stable as well as provide estimates of the mean and standard deviation.

Once failure data have been collected, it is often useful to display them in a graphical format. Key reliability metrics include:

- *Probability density function* (PDF) can be plotted as a histogram showing the number of failures in each time block
- The *hazard function* shows the failure rate as a function of time:

$$\lambda(t) = \frac{Fraction\ of\ failures\ during\ the\ time\ period}{Amount\ of\ time\ during\ the\ period}$$

- The *reliability function* plots reliability as a function of time:

$$\mathrm{R}(t) = \frac{Number\ surviving\ at\ the\ end\ of\ the\ period}{Number\ of\ units\ tested}$$

The Weibull reliability function is,

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

5. Failure Analysis Methods

It is important to have as much information as possible about each failure.

The first step in failure analysis is to determine why the item has been classified as failed.

If the item did not physically fail, thorough study of the environment in which it functioned is still required.

If the part has become deformed or broken, 2 questions arise:

1. Has the part failed to meet designed strength?
2. Has the part been subjected to stress beyond its designed strength?

If the answer to Q2 is yes, then,

- Is the design strength adequate?
- Was the part applied inappropriately?
- Did the environment (thermal, chemical, electromagnetic, vibration, etc.) contribute to the failure?

The reliability program should support a traceability system that will identify materials, methods, processes and supplier information for each item. This will help determine whether other items in the field are vulnerable.

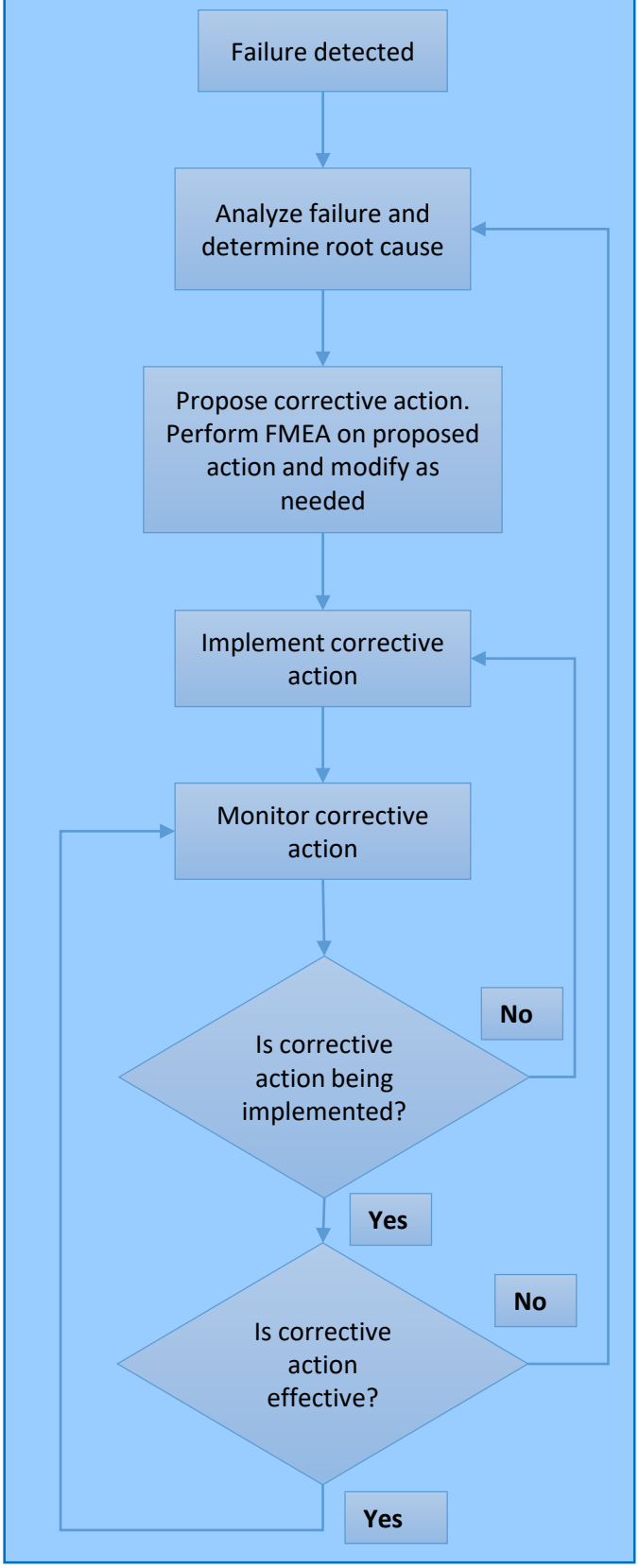
After determining the root case, a report should be generated that includes recommendations for changes that will help reduce the reoccurrence.

6. Failure Reporting, Analysis and Corrective Action System (FRACAS)

FRACAS provide an organized disciplined approach to dealing with unanticipated failures. The first step is to establish a *failure review board*, who are responsible for implementing the following guidelines:

- Require prompt and complete failure reporting
- Begin the reporting with the earliest tests of products and processes, and continue through the life of the product
- The analysis and corrective action phase of each report must determine root causes and appropriate corrective/preventive action
- The corrective action must be closed loop. Implement audits to determine whether corrective actions have been taken and whether they are effective. When corrective actions do not resolve the problem, the failure review board is responsible for a restudy of the failure mechanism and implementation of additional corrective actions.

FRACAS flowchart



Part 4A: Planning

1. Reliability Test Strategies

Highly Accelerated Life Testing (HALT): performed on the preliminary design prototypes. The strategy for this reliability testing program is to stress the units beyond the design limits in order to cause failures and hence design out weaker components. Results from HALT testing cannot be used to estimate reliability since the units are stressed beyond design limits.

Life Tests: performed on production units; accumulate test time on several units while recording failures to estimate the failure rate or MTBF. Test time may be lengthy.

Highly Accelerated Stress Screening (HASS): should be performed before the product is delivered to the customer. The strategy is to stress the entire production run to eliminate defective product from the population and to detect any shift in the production process. Limits early-life failures.

Compliance Testing: verify that the product conforms to some minimum reliability value. Accumulates time on several units. Not for product improvement.

Steps in developing a reliability test plan (Kececioglu, 1993):

1. Determine the test requirements & objectives
2. Review existing data to determine if any requirements can be met without testing
3. Review the list of tests to determine whether combining any tests would be economically feasible
4. Determine the necessary tests
5. Allocate the resources necessary to perform the tests
6. Develop test specs and data handling & storage procedures. Review acceptance and qualification criteria. Establish procedures for making future changes in the test specs.
7. Assign the responsibility for conducting the tests, analyzing the results, and providing the overall integration of the testing program
8. Develop forms & procedures for reporting test results
9. Develop procedures for maintaining the test status information throughout the entire testing program

Four major reliability test categories classified by the phase of the development/production cycle are:

Product Development Tests: performed with the intent of taking action to improve the design in the event of failures and to evaluate the system design. Can demonstrate the functional capabilities of the product but cannot determine the failure rate/MTBF.

Reliability Performance Tests: or "reliability qualification tests". Performed after design is complete to demonstrate the system can meet the specified requirements under the specific operating conditions.

Reliability Acceptance Tests: conducted during production phase. Demonstrates that the reliability has not been compromised during the production process.

Reliability Verification Tests: these tests are performed to show compliance to stated reliability parameters such as MTBF or failure rate. Formal, statistical, and requires standard procedures.

One-shot items: units that perform successfully or fail when they are called on to operate, e.g. sensors or fuses.

Reliability tests can be continuous or they can be pass/fail. Results for continuous tests are recorded as variables data, and the results of pass/fail tests are recorded as attribute data.

Before beginning a test, it is necessary to decide how the test will be terminated:

Truncation: terminating the test at some specified amount of time or other stress metric.

Test-to-Failure: continue tests until all units have failed.

Truncated test-to-failure: test a sample of items until a certain number have failed (the truncation point).

Degradation test: terminates when a product reaches a predetermined reduced level of performance.

Reliability Growth Testing

Reliability growth is the improvement in product reliability over a period of time, due to changes in the product design.

Test, analyze and fix (TAAF): test new design, identify other problems and eliminate these (redesign), repeat.

MIL-HDBK-189 provides an understanding of the concepts and principles of reliability growth testing.

Frequently used models to track reliability growth:

Duane model: based on observation & uses a deterministic method to track reliability growth.

Army Materiel Systems Analysis Activity (AMSAA) model: allows for statistical procedures to be used.

Both models use the observed linear relationship between the log of cumulative MTBF & cumulative test time.

The data are accumulated as the TAAF process progresses.

The cumulative MTBF is calculated:

$$\theta_m = \frac{T}{r}$$

Where,

T = total unit test time inc. all testing

r = total number of failures inc. all testing

In the Duane model, $\theta_m = K(T)^b$

As the TAAF process continues and T increases, θ_m will increase, reflecting reliability growth. This can be tracked:

$$\log\theta_m = \log K + b \log T$$

This is a linear equation in logarithms and will plot as a straight line on log-log graph paper.

The reliability growth rate *b* can be measured from the graph or calculated:

$$b = [\log(\theta_1/\theta_0) / \log(T_1/T_0)]$$

Where θ_0 = initial cum. MTBF at test time T_0; θ_1 = cum. MTBF after test time T_1.

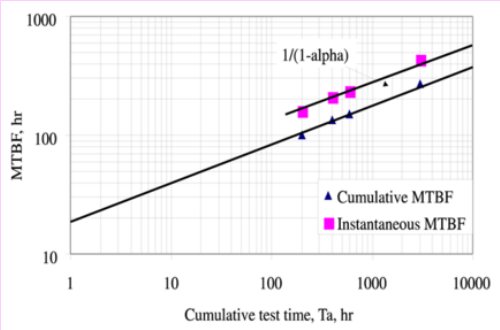
In the Duane model, the true MTBF will grow at the same rate as the cum. MTBF value. At any time during the testing,

CRE Part 4: Reliability Planning, Testing and Modeling

the true MTBF (θ) can be found as,

$$\theta = \theta_m / (1 - b)$$

A growth rate of ~0.25-0.4 is average for most projects. A higher growth rate shows the efforts to eliminate design weaknesses has been given top priority. A lower rate indicates that reliability improvement actions are taken to eliminate only the most obvious design flaws.



2. Environmental and Conditions of Use Factors

Use Factors are those environmental conditions that are normal for the product.

Stresses are those conditions that are outside the design ranges.

Understanding the use environment and use condition of a product is a prerequisite for designing effective reliability tests. The purposes for characterizing environmental and use factors are:

- To design a product that meets customer expectations
- To determine product safety and durability
- To plan an effective reliability test that can provide information for reliability prediction
- To evaluate the product transportation, storage, installation and use condition
- To design product monitoring schemes
- To effectively mitigate or reduce stress effects

Environmental and other stress factors which can negatively affect the reliability of a product:

Temperature	Vibration	Humidity	Rust
Corrosion	Lightning	Voltage	Salt
Dust	Chlorine	RF Interference	

3. Failure Consequence

Consequences of failure must also be examined. These are typically costs to the customer and ultimately the producer. Reliability acceptance goals will limit failure occurrence and consequent costs. Acceptance criteria will be:

- Functional, Environmental & Time requirements

Testing should *validate* that these goals will impact failure rates and consequences.

Reliability goals impose specifications on the product. Testing will *verify* that these specs are being met.

Tools: Delphi method, RCA, FMEA, probability/impact matrix

4. Failure Criteria

Understanding product failures and failure consequences will lead to a proper definition of product failure criteria – what is defined as a success or failure? The acceptance criteria fall into 3 categories:

1. Functional requirements (design)
2. Environmental requirements (use conditions)
3. Time requirements (expected lifetime)

The required performance must be quantifiable and measurable.

Both environmental and time requirements should be connected to the company's warranty program.

5. Test Environment

The strategy of the reliability test and the product itself will determine the environment of the test, which will reflect the environment of use. Increased environmental stress can be used to accelerate these tests. Test chambers capable of the appropriate test environments are required.

Combined Environmental Reliability Testing (CERT): an application of multiple environmental stresses simultaneously, e.g. temperature cycling and vibration. Test equipment is now available that will allow the simultaneous application of multiple environments.

The results of the testing should be both accurate and precise. *Accuracy* implies that the data are on target, and *precision* relates to the tolerance limits of the equipment.

Part 4B: Testing

1. Accelerated Life Tests (e.g., single-stress, multiple-stress, sequential stress, step-stress)

Accelerated life testing can be employed to reduce the time required for reliability testing. The strategy is to increase the rate at which failures occur, not cause new types of failures.

Single-stress tests: some devices can be accelerated in time by simply using them at a rate that exceeds the normal use rate. Increase stress is often used to accelerate time. Suppose under normal use, 5% of the population will fail in time t_n. If the units are operated under an increased environmental stress, the same 5% will fail in time t_s where $t_s < t_n$. the ratio t_n / t_s is the *acceleration factor* A_F. The equivalent time of the test is $t_{Eq} = A_F \times t$

The models used to accelerate time will depend on several factors. Two commonly used models are the *Arrhenius model*, when temperature is used to accelerate time, and the *power law* (or inverse power law) model when stresses other than temperature are used. The acceleration factors are usually derived from the models using physics-of-failure analysis for the various failure modes, testing to determine the value of the acceleration factor, and then verifying the acceleration factor. This can be lengthy and costly.

The Arrhenius Model

The Arrhenius acceleration model is based on the Arrhenius equation, which states that the rate of a chemical reaction increases as the temperature increases:

$$R = Ae^{\frac{-E_A}{KT}}$$

where,
R = reaction rate of the chemical reaction
A is a scaling factor
K = Boltzmann's constant (8.617×10^{-5} eV/degK)
T = temperature in Kelvin
E_A = activation energy in eV

Reaction rate can be thought of as being synonymous with failure rate. Increasing the test temperature will increase the constant failure rate of the units, which has the effect of accelerating time.

The ***acceleration factor*** is the ratio of the reaction rate at the increased temperature T_S and the reaction rate at the use temperature T_U:

$$A_F = \frac{R_S}{R_U}$$

This reduces to,

$$A_F = e^{\frac{E_A}{K}\left(\frac{1}{T_U} - \frac{1}{T_S}\right)}$$

The key to using the Arrhenius model is determining the appropriate activation energy. An approximation is that for every 10degC increase in temperature, the failure rate doubles. This can be verified by testing units at various stress levels – at low, medium, and high values, and analyzing failures to determine the acceleration factor.

The Power Law Model

The power law model applies to units subjected to accelerating stresses that are not thermal. The law states that the life of the product is inversely proportional to the increased stress:

$$\frac{life\ at\ rated\ stress}{life\ at\ accelerated\ stress} = \left[\frac{accelerated\ stress}{rated\ stress}\right]^b$$

Testing at 2 stress levels will give data to solve for b, which is then assumed to remain constant over the range of applied stress.

Multiple-stress tests: the Eyring model, the Arrhenius model and the power law model can be combined to be used with multiple stresses.

Eyring model: An accelerated life testing model based on quantum mechanics that is typically used when temperature or humidity is the accelerated stress.

Step-stress testing: sometimes used to further accelerate a reliability test. Involves increasing the stress level at fixed points and recording the failures or degradation levels.

Highly Accelerated Life Testing (HALT): continue to increase stress on a unit until failure occurs. The intent is to identify the weak link in the design in order to make a change to improve the design. There are 3 distinct phases of testing:

Phase 1: Pre-HALT Phase
Document test procedures. Prepare test equipment, resources, and procedures for recording of data.

Phase 2: HALT Phase
Perform testing. Monitor, analyze and record results.

Phase 3: Post-HALT Phase
Perform root cause analysis on the test failures and determine corrective action.

Margin tests apply increasing stress levels in an effort to develop a margin above spec for a particular failure mode.

2. Stress Screening

Stress screening is a final test performed on production units prior to release to the customer.

Burn-in testing: allows the product to accumulate use time before it is delivered, to reduce the occurrence of early-life failures after delivery.

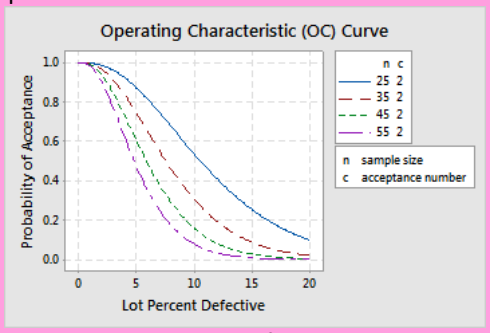
Environmental Stress Screening (ESS): to accelerate the rate at which the weak units fail, some stress (usually environmental) is applied during the test. This stress should not damage an acceptable unit.

Highly Accelerated Stress Screening (HASS): a stress screening program to ensure the final product will exceed its environmental requirements. The stresses may exceed the design spec. The design has been proven to be robust using a HALT program. The purpose of HASS is to detect a shift in a process that has previously been shown to be capable. To use it successfully, accepted process quality

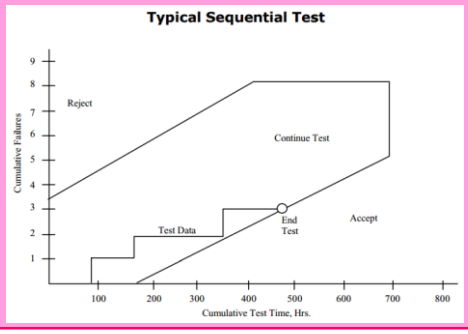
procedures must be applied to the process, and the process must have capability measures.
Highly Accelerated Stress Audit: can replace HASS. Involves stress screening just a sample of production units. Will detect a process shift, but risks allowing some early-life failures to reach the customer.

3. Qualification/Demonstration Testing

Qualification/demonstration reliability testing is referred to as *compliance testing*. These are used to demonstrate that a product parameter (e.g. MTBF) conforms to a given requirement. References:
IEC Standard 61124-2006; 61123-1997; MIL-HDBK-781.
The assumption for all compliance test plans that use time as the continuous variable is that the failure model is the exponential, implying a constant failure rate and that MTBF/MTTF = 1/failure rate. The assumed failure model for the fixed-trial/failure test plans is the binomial.
All compliance test plans are described by an ***operating characteristic curve*** – a graph showing the probability of demonstrating compliance given the true value of the product parameter.



Fixed-Time Test Plans: consist of placing units on test to accumulate test time (T) while recording failures. The test is continued until either the required amount of test time has been accumulated (accept) or the allowable number of failures, *c*, has been exceeded (reject). The decision to accept cannot be made until the test is completed.
Sequential Test/Probability Ratio Sequential Test Plans: results are continuously assessed to arrive at 1 of 3 alternative decisions – accept, reject or continue testing, in the minimum amount of test time. The graph boundaries are defined by the min & max acceptable MTBF/failure rates.



4. Degradation

It may be necessary to test units to determine the end of useful life. Some form of acceleration (e.g., rapid cycling, temperature, salt spray) to increase the rate of wear is necessary.
It is sometimes possible to extrapolate to a failure time by observing the amount of degradation a unit has experienced in a given time, hence reducing test time. Examples of models used to model the rate of degradation are the linear, exponential and power law models. The model could be derived using physics of failure.

5. Software Testing

White-box/structural testing: the tests are designed and implemented with complete knowledge of the inner structure of the system being tested. The intent is to cover all paths and to test all branches within the system. Focuses on how the software works.
Fault Injection: to seed the program with known faults and measure the test time to uncover a given % of these faults. Used to predict the amount of testing required.
Many s/w faults are due to poorly stated/understood specs.
Black-box/functional testing: testing to external specs, to determine if the software responds as expected.
Beta testing: involve customers in testing before release.
Operational profile testing: exercises the software package under all the anticipated conditions of the users.
Regression testing: selective retesting to detect earlier solved faults that did not cause unintended behavior of the software or other faults: verify if the fault is solved, verify if previous solved problems are still fixed, and test if the software doesn't have unwanted side effects due to the fix.
Field testing: testing in the customer final environment
Random-input/monkey testing: random entering of possible inputs to the software
Boundary value testing: testing at the boundaries of the software input values
Automated software testing: writing another software routine replacing the users and playing them back to be able to stress the software routines automatically.

Part 4C: Modeling

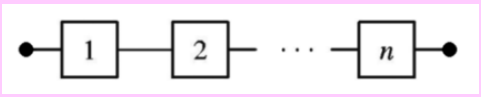
1. Reliability Block Diagrams and Models

A math model reduces the system to a graphical representation of the interconnections of its subsystems, hence the system reliability can be modeled using the reliability of the various subsystems.

Advantages: predicting system reliability; identify weak links in the system for targeted reliability improvements; determine test and maintenance procedures.

Static System Reliability Models

Series System: there is only one path for system success. If any subsystem fails, the system will fail. Note that system interfaces may also be a source of failure.

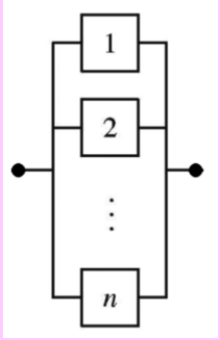


Assume all the probabilities of failure for the individual subsystems are independent. The system reliability for mission time, t, for n subsystems, is

$$R_{system}(t) = R_1(t) \times R_2(t) \times R_3(t) \times \ldots \times R_n(t)$$

Note that the reliability of the system will be less than the reliability of any subsystem. For the largest reliability improvement, target the subsystem with the lowest reliability.

A series system will require minimum parts, consume minimum power and hence dissipate less heat, take up less room, add less weight, and be cheaper to build.



Parallel System: Redundancy can be designed into a system to increase system reliability. A parallel system provides more than 1 path for system success. Beware of single-point failures bringing down all paths simultaneously. A parallel system will fail only is all subsystems fail:

$$R_{system}(t) = 1-[1-R_1(t)] \times [1-R_2(t)] \times [1-R_3(t)] \times \ldots \times [1-R_n(t)]$$

The reliability of the system will be greater than the reliability of any of the redundant subsystems.

Series-Parallel Model: a system may be modeled as a combination of series and parallel subsystems. System reliability can be found by converting the system to an equivalent series or parallel model.

Special Case Reliability Modeling

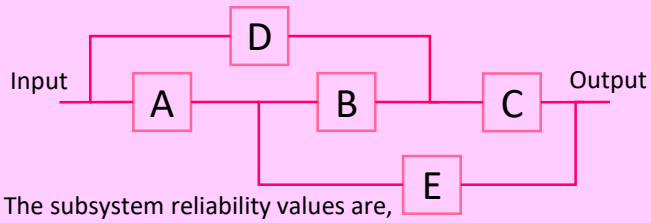
m out of n System Model: this is a parallel system of n equivalent subsystems. System success requires that at least m of the subsystems will not fail. The system reliability is,

$$R_{system}(t) = \Sigma C_i^n (R)^i (1-R)^{n-i}, for\ i = m\ to\ n$$

$$C_i^n = \frac{n!}{[i!\,(n-i)!]}$$ is the number of combinations of *n* items taken *i* at a time.

Significance: all engineers need to know how to express results using correct significance (decimal places).

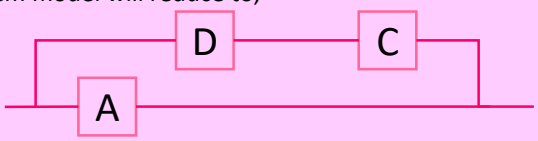
System Model Using Bayes's Theorem: used when systems cannot be reduced to a series or parallel model. Perform Bayesian analysis. Example: find the reliability of the system



The subsystem reliability values are,

$$R_A = R_B = R_C = 0.95;\ R_D = R_E = 0.99$$

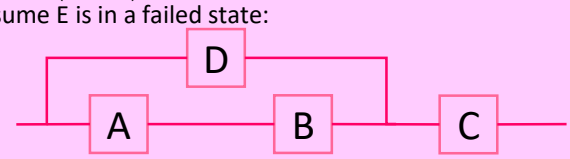
Choose subsystem E. Assume E is in a success state. The system model will reduce to,



And the system reliability with E good is,

$$R_{(E\ success)} = 1 - [1-(0.99)(0.95)][1-0.95] = 0.9970$$

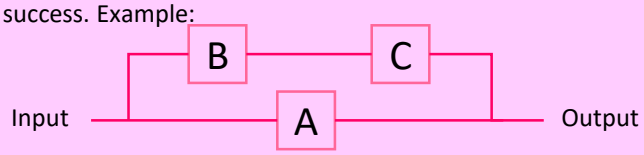
Assume E is in a failed state:



$$R_{(E\ Failed)} = [1-(1-0.99)(1-(0.95)(0.95))][0.95] = 0.9491$$

$$R_{system} = R_{(E\ success)} \times P(E\ success) + R_{(E\ Failed)} \times P(E\ failed) = 0.9965$$

Truth Table Method: finds system reliability via a systematic method to identify all the events that will result in system success. Example:



Subsystem reliabilities are, $R_A = 0.85$, $R_B = 0.90$, $R_C = 0.92$

The total number of events is $2^3 = 8$

Construct a truth table using S = success, F = failure:

Event #	Subsystem A	Subsystem B	Subsystem C	System	Event probability
1	S	S	S	S	(.85)(.90)(.92)=0.7038
2	S	S	F	S	(.85)(.90)(.08)=0.0612
3	S	F	S	S	(.85)(.10)(.92)=0.0782
4	S	F	F	S	(.85)(.10)(.08)=0.0068
5	F	S	S	S	(.15)(.90)(.92)=0.1242
6	F	S	F	F	
7	F	F	S	F	
8	F	F	F	F	

The system reliability is the sum of the probabilities of the events that result in system success = 0.9742.
This can be verified using the series-parallel model.
NOTE: the solutions for system reliability for all above static math models are distribution free.

Series Model (Constant Failure Rate): assume that the failure distribution for each of the subsystems is exponential. Each subsystem *i* has a constant failure rate λ_i. The system failure rate is also constant and equal to the sum of the subsystem failure rates:

$$\lambda_{System} = \sum_{n} \lambda_i$$

The system reliability is,

$$R_{System} = e^{-(\lambda_{System})^t}$$

The MTTF/MTBF of the system is,

$$\theta = \frac{1}{\lambda_{System}}$$

Dynamic System Models: dynamic math models are time dependent. To analyze these, assume a failure distribution for the subsystems. For the following models, assume the exponential distribution and a constant failure rate for the subsystems.

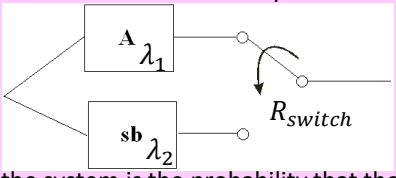
Load Sharing Model: the subsystems of an active parallel system are connected such that each shares equally in the total load. The subsystems are derated so that each is operating at less than its max load capacity → failure rate is lowered, reliability improves. If a failure occurs, the remaining subsystems have enough capacity to carry the system load, but they will be operating at a higher stress. For the case of 2 units in a load-sharing configuration, each subsystem operates with a constant failure rate λ_1, as long as both are operating. If one subsystem fails, the remaining subsystem will continue to operate but at an increased failure rate $\lambda_2 > \lambda_1$. System success for a mission time *t* would be described as both subsystems operating successfully for the entire time *t* at failure rates = λ_1, or a failure occurring at time $t_1 < t$ and the remaining subsystem operating for a time of $t - t_1$ at a failure rate of λ_2:

$$R_{System(t)} = e^{-2\lambda_1 t} + \frac{2\lambda_1\left(e^{-\lambda_2 t} - e^{-2\lambda_1 t}\right)}{2\lambda_1 - \lambda_2}, \text{ if } 2\lambda_1 - \lambda_2 \neq 0$$

If $2\lambda_1 = \lambda_2$ the denominator of the above equation = 0 and the equation reduces to,

$$R_{System(t)} = e^{-2\lambda_1 t} + 2\lambda_1 t e^{-\lambda_2 t}$$

Standby Redundant Systems: a system that has parallel units that are utilized only in the event of a failure.



The failure rate of the primary and secondary units is λ and the mission time is *t*. The reliability of the system is the probability that the primary unit will operate successfully for time *t*, or the primary unit will fail at $t_1 < t$, and the secondary unit will operate successfully for the time $t - t_1$.

$$R_{System(t)} = e^{-\lambda t}(1 + R_{switch}\lambda t)$$

Or,

$$R_{System(t)} = e^{-\lambda_1 t} + R_{switch}\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

2. Physics of Failure (PoF) and Failure Mechanisms

The physics-of-failure approach to reliability uses data on the actual environmental conditions and loading during the life of the unit to make predictions about the unit's current "health" and its expected reliability. PoF uses knowledge of basic failure processes to prevent failures through robust design and manufacturing practices and aims to:

- Design-in reliability up front
- Eliminate failures prior to testing
- Increase fielded reliability
- Promote rapid, cost-effective deployment of health and usage monitoring systems (HUMS)
- Improve diagnostic and prognostic techniques and processes
- Decrease operation and support costs

PoF Reduces lifecycle costs by,

- Generating alarms for impending failures
- Reducing unscheduled maintenance
- Extending preventive maintenance times
- Reducing inspection costs & replacement part inventory

2 approaches to physics-of-failure:

Model-based Approaches:

These systems attempt to derive a mathematical model to describe unit performance and to generate reliability predictions. Disadvantage: detailed knowledge about the geometry and composition of the unit are needed but not always available.

Failure modes

Failures in engineering design and systems usually occur in the 4 forms: hardware failures, software failures, human failures and organizational failures.

Failure modes & models for circuit boards:

Failure Sites	Failure Modes	Failure Models
Fracture	Die attach, wirebond, solder leads, bond pads, traces, interfaces	Nonlinear power law
Corrosion	Metalizations	Eyring
Electro migration	Metalizations	Eyring
Conductive filament migration	Between metalizations	Power law
Stress-driven diffusion voiding	Metal traces	Eyring
Time-dependent dielectric breakdown	Dielectric layers	Arrhenius

CRE Part 4: Reliability Planning, Testing and Modeling

Failure modes most commonly seen in machine and machine parts:
Force and/or temperature-induced elastic deformation, yield, brinelling, ductile rupture, brittle fracture, fatigue, corrosion, wear, impact, creep, thermal relaxation, stress rupture, thermal shock, galling and seizure, spalling, radiation damage, buckling, creep buckling, combined creep and fatigue.
Data-Driven Approaches: relies on a software system that learns the data patterns that lead to failure. Disadvantage: historical data must be available to establish correlations and detect patterns.
Physics-of-failure is associated with *prognostics and health management* (PHM), where "health" refers to a methodology for continuous assessment of the remaining useful life of a unit.

3. Failure Models

Common Life-Stress Models:

- ***Arrhenius***: a single stress model used when temperature is the accelerated stress
- ***Eyring***: a single stress model used when temperature or humidity is the accelerated stress
- ***Inverse power law*** (IPL): a single stress model typically used with a nonthermal stress (e.g., cyclic loading, vibration, voltage or temperature cycling)
- ***Temperature-humidity***: a double Arrhenius model used when temperature and humidity are 2 acceleration variables
- ***Temperature-nonthermal***: a combination of the Arrhenius and IPL relationships used when 1 stress is temperature and the other is nonthermal (e.g., voltage)

Mathematical equations of life versus stress are:

- Linear: $Life = a(Stress) + b$
- Exponential: $Life = be^{a(Stress)} or\ Life = Ae^{\frac{E_a}{kT}}$
- Power: $Life = b^{(Stress)a} or\ Life = \frac{1}{k(Stress)^n}$
- Logarithmic: $Life = a * \ln(Stress) + b\ or$
  $Life = e^{a_0 + a_1 Stress_1 + \cdots + a_n Stress_n}$
- Combinations of these forms as well:

$$Life = \frac{1}{(Stress)} e^{-(A - \frac{B}{Stress})}$$ Eyring relationship

Arrhenius Temperature Acceleration Model

See 4B1 for notes on the Arrhenius Model. The Arrhenius model predicts how time-to-fail varies with temperature. This model is widely used for electronics, chemicals and packaging industries.

Inverse Power Law for Voltage Acceleration

Let T(volt) and T(voltu) be the failure times that would result for a particular unit tested at increased voltage and use-voltage conditions, respectively:

$$T(volt) = \frac{T(volt_u)}{AF(volt)} = \left(\frac{volt_u}{volt}\right)^{\beta_1} T(volt_u)$$

So,

$$T(volt) = \frac{\alpha}{(volt)^{\beta}}$$

The relationship is known as the inverse power law because generally, $\beta_1 > 0$. step-stress testing is sometimes used to further accelerate a reliability test. This technique involves increasing the stress level at fixed points and recording the failures or degradation levels.

Degradation Model

Two common assumptions are typically made when degradation data are modeled:

1. A parameter D, which can be measured over time, drifts monotonically (up or down) towards a specified critical value DF (degradation failure). When it reaches DF, failure occurs.
2. The drift, measured in terms of D, is linear over time with a slope R, which depends on the relevant stress the unit is operating under and also the (random) characteristics of the unit being measured.

The central idea is to model unit degradation paths to predict unit times to fail, then use the characterized times-to-failure to estimate reliability.
This can be more accurate than traditional MLE because it includes information about how close to failure an item may be.
For the case when D=0 at time 0 and the degradation failure is the same for all units models can be fit using all the degradation readings and linear regression.

The Eyring Model

Can be used to model acceleration when many stresses are involved. The model includes temperature and can be expanded to include other relevant stresses.

$$t_f = AT^{\alpha} exp\left[\frac{\Delta H}{kT} + \left(B + \frac{C}{T}\right) S_1\right]$$

Advantages of the Eyring model:

- Can handle many stresses
- Can be used to model degradation data as well as failure data
- The ΔH parameter has a physical meaning and has been studied and estimated for many well-known failure mechanisms and materials

Disadvantages:

- Even with just 2 stresses, there are 5 parameters to estimate. Each additional stress adds 2 more unknown parameters
- Many of the parameters may have only a second-order effect
- The form in which the other stresses appear is not specified by the general model and may vary according to the particular failure mechanism

CRE Part 4: Reliability Planning, Testing and Modeling

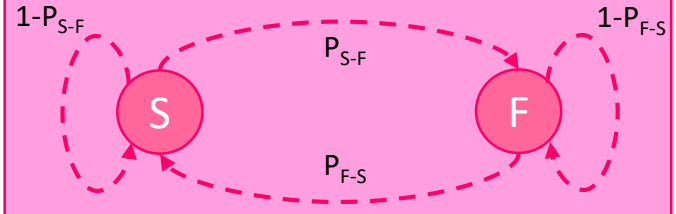
Proportional Hazard Models
These are models having the property that individual component hazard rate functions are proportional to each other.

4. Reliability Prediction Methods

Modeling a dynamic system to predict its performance can become complex. To analyze these systems, a simulation technique can be used. To use simulations, the defining parameters of each distribution must be known.

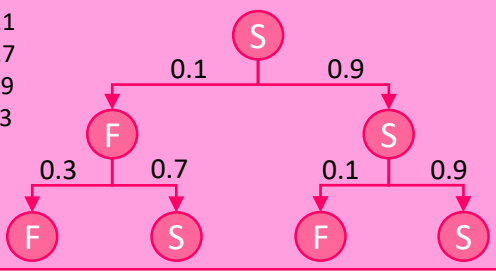
Monte Carlo: Monte Carlo simulation performs risk analysis by building models of possible results by substituting a range of values—a probability distribution—for any factor that has inherent uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending upon the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete. Monte Carlo simulation produces distributions of possible outcome values.

Markov Analysis: a complex system can be analyzed as a Markov process. This can be used to find the probability of being in a give state at some time in the future if the probability of moving from one state to another state is known and the probability remains constant. The system can exist in only one state at a time, and except for the immediately preceding state, all future states are independent of past states.



P_{S-F} is the probability of transition from state S (success) to state F (failure) in a given time interval.
P_{F-S} is the probability of transition from state F to state S in the same time interval.
$1-P_{S-F}$ is the probability of remaining in state S if the system is in state S.
$1-P_{F-S}$ is the probability of remaining in state F if the system is in state F.
A tree diagram can be used to model a simple system:

$P_{S-F} = 0.1$
$P_{F-S} = 0.7$
$P_{S-s} = 0.9$
$P_{F-f} = 0.3$



A Markov analysis will result in a transitional probability matrix that can be evaluated to find the state probabilities over many periods of time.

$$T = \begin{bmatrix} 1-PS_{-F} & P_{S-F} \\ P_{F-S} & 1-PF_{-S} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}^2 = \begin{bmatrix} 0.88 & 0.12 \\ 0.84 & 0.16 \end{bmatrix}, where\ time\ interval = 2$$

Part Count Predictions & Part Stress Analysis

Reliability prediction should be used early in the design stage. Many times, the prediction is done without any actual reliability data on the particular system, although there may be data on some of the components. The reliability prediction can be used to:

- Determine the feasibility of the design to meet the system reliability goals
- To focus attention on the weak links in the design
- To asses the impact of design changes on system reliability
- To compare the reliability of competing designs early in development stage
- To assist in establishing maintenance procedures

MIL-HDBK-217 provides predictive data for electrical/ electronic systems. This handbook provides base failure rates for components, which can be multiplied by "π (pi) factors" such as environmental stress, quality control, etc.

Part count method: when a circuit board (subsystem) failure rate is predicted by adding the predicted failure rates of all the components on the board. Assumes a series model; worst case scenario.

PRISM reliability prediction method: developed by the Reliability Analysis Center (RAC) in an attempt to overcome some of the deficiencies of MIL-HDBK-217. uses a combination of both additive and multiplicative π factors. PRISM also allows for consideration of early-life failures as well as failures due to a constant failure rate.

217(plus) method: an outgrowth of PRISM. Contains new part type rate models including connectors, switches, relays.

The *Handbook for Reliability Prediction for Mechanical Systems* (NSWC-07) contains base failure rates for mechanical components, which can be adjusted using multipliers of "c" factors depending on the type of material, physical configuration, heat treatment, the use environment, etc.

Non-electronic Parts Reliability Data (NRPD-2011) contains failure rates for mechanical and electromechanical parts. This method considers failures due to wear out.

Telecordia SR-332 has prediction data in tables and graphs for communication equipment with commercial applications.
The *Government-Industry Data Exchange Program* (GIDEP) is a source available to military contractors that contains failure reports on commercially available subsystems such as motors, compressors, pumps.

Reliability Prediction Methods

The reliability of a product needs to be continually evaluated using the available information during all stages of design, development and production, to assure that the product will meet system reliability requirements.
Early reliability predictions: useful, but based on simplifying assumption that each component has an inherent constant failure rate. Cannot take into account the human factors that most likely cause failure. Does not have relevant experimental data for support.
If testing results are available, reliability predictions can be made based on the distribution of times to failure and the estimated parameters. If the exponential distribution models the times to failure, the estimated parameter is the mean → this is the mean time to failure (MTTF, θ). If *n* units are tested, and *r* units fail, MTTF is,

$$\hat{\theta} = \frac{T}{r}$$

where,
T = total time accumulated on all units
r = number of failures

The estimated failure rate is the reciprocal of the mean:

$$\hat{\lambda} = \frac{r}{T}$$

The estimated reliability for a mission time of *t* is,

$$\hat{R} = e^{-(\frac{t}{\theta})} = e^{-\lambda t}$$

For a **repairable** system, if the exponential distribution model is appropriate, the above is valid up to the first failure. After the first failure, some parts of the system may be restored, hence the system comprises subsystems and components that have acquired different operating times. This means the system cannot be modeled using a constant failure rate. Rather than making predictions of the reliability of a system that has experience several repair cycles, predictions should be made as to the number of spare parts needed in order for the system to meet specified availability requirements.

5. Design Prototyping

Prototyping is the process of rapidly putting together a working model (a prototype) in order to test various aspects of a design, show ideas or features and gather early user feedback.
A key concept is that prototyping is *user-centered design*. Prototypes allow designers to spot problems early, examine their product, and run through reliability tests.

Rapid prototyping refers to physical objects that are automatically constructed with the aid of additive manufacturing technology. Rapid prototyping systems use liquid, powder or sheet materials to form physical objects. It takes the virtual designs from CAD or animation modeling software (AMS). These designs are further transformed into thin, virtual, horizontal cross-sections, and then the process of creating successive layers continues until the model is complete. The layers are then joined to yield the final shape.
Rapid prototyping technologies include:

1. Selective laser sintering
2. Fused deposition modeling
3. Stereolithography
4. Laminated object manufacturing
5. 3D printing

Rapid prototyping is becoming more capable (especially with additive methods for solid parts) and is already producing large products. It is therefore becoming “rapid manufacturing”.

CRE Part 5: Lifecycle Reliability

Part 5A: Reliability Design Techniques

1. Design Evaluation Techniques (V&V)

The Reliability Engineer performs evaluation functions in each step of the design process:

Concept: preliminary estimates of reliability requirements should be made

Design Team Effort: provide guidance on design options, data on component/sub-system/system reliability, review potential failure modes and failure prevention methods (fault tolerance or fault avoidance).

Design Review: validation – test data showing product meets reliability requirements

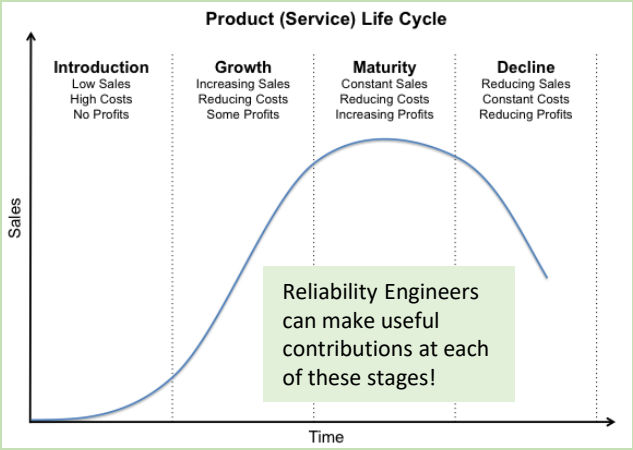
Preproduction: review manufacturing process options

Production: verification – establish testing program to assure production output meets requirements

Post-Production: failure studies (FRACAS: failure reporting analysis and corrective action system)

4 types of Evaluation:

1. Environmental Stress Screening/accelerated life tests
2. Reliability development/growth tests (series of tests)
3. Reliability qualification tests (on a production sample)
4. Product reliability acceptance tests (periodic)



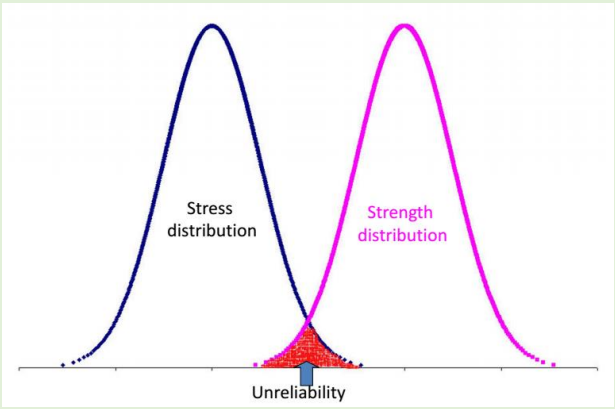
2. Stress-Strength Analysis

Failure of a part will occur when STRESS > STRENGTH of the part for that stress. A good design incorporates safety factors/margins to ensure strength > stress.

$$\mu_{Strength} > \mu_{Stress}, where\ \mu = mean$$

$$Safety\ Factor = \frac{\mu_{Strength}}{\mu_{Stress}} > 1$$

When viewed as distributions, there can be an interference region in which it is possible for stress to exceed strength → FAILURES.



The ***difference distribution*** can be used to solve for the probability of failure:

$$\mu_{Difference} = \mu_{Strength} - \mu_{Stress}$$

$$\sigma_{Difference} = \sqrt{\sigma_{Strength}^2 + \sigma_{Stress}^2}$$

The probability of failure is the area to the left of 0 in the difference distribution. It is the area from the standard normal tables to the left of a z value equal to:

$$z = \frac{(0 - \mu_{Difference})}{\sigma_{Difference}}$$

4 ways the designer can improve reliability:

- Increase mean strength: use different materials/design
- Decrease strength variation in materials/process
- Decrease the mean stress: control the loading
- Decrease the stress variation: limit the use environment

3. Design of Experiments (DOE)

Experimental Error: the variation in the response variable when levels and factors are held constant. The accurate determination of experimental error and cost are competing design properties.

Factor/Variable: an assignable cause that may affect the responses, and of which different levels are included in the experiment.

Levels: the possible values of a factor in an experimental design.

Noise Factors: those factors that aren't controlled in an experiment.

Replication: the repetition of the set of all the treatment combinations to be compared in an experiment. Each of the repetitions is called a *replicate*.

Response Variable: the variable that shows the observed results of an experimental treatment.

Treatment: a combination of the levels of each of the factors assigned to an experimental unit.

The objective of a designed experiment is to generate knowledge about a product or process. The experiment seeks to find the effect a set of independent variables has on a set of dependent variables (y=f(x); x = independent/ input variables, y = dependent/output variable). The independent variables that the experimenter controls, are the *control factors*.

A **FULL-FACTORIAL** experiment tests all possible combinations of levels and factors, using one run for each combination. The formula for number of runs is,

$n = L^F$, where L = number of levels; F = number of factors.

Planning and Organizing Experiments

First consideration: What question are we seeking to answer? This is the experimental objective.

Then, establish an appropriate measurement system. The *Rule of 10* measurement principle says that the finest resolution of the measurement system must be less than or equal to 1/10 of the tolerance.

Then, select the factors and levels (by asking Experts).

Next, choose the appropriate design. This may be constrained by affordability, time, etc. Typically, 20-40% of the budget should be allocated to the 1st experiment as it seldom meets the objective and raises questions for further studies.

Randomization

The tests should be performed in random order to spread out the variation caused by noise variables. There are 2 ways to randomize tests:

1. Number the tests and randomize those numbers to obtain the order in which tests are performed. This is a *completely randomized design*.
2. Randomize the run order, but once a run is set up, make all the replicates for that run.

#1 is better, however it may require more time & effort.

Blocking

With a *randomized block design*, the experimenter divides subjects into subgroups called blocks, such that the variability within blocks is less than the variability between blocks. Then, subjects within each block are randomly assigned to treatment conditions.

Main Effects

The first step in calculating the main effects is to average the results for each level of each factor, then plot the results. The *main effect* is the high-level result minus the low-level result for the factor. Therefore, the larger the absolute value of the main effect, the more influence that factor has on the output variable. If experimental error is large, ANOVA can be used:

Analysis of variance (**ANOVA**) is a collection of statistical models used to analyze the differences among group means and their associated procedures (such as "variation" among and between groups).

Beta risk is the probability that a false null hypothesis will be accepted by a statistical test. This is also known as a Type II error. The primary determinant of the amount of beta risk is the sample size used for the test. The larger the sample tested, the lower the beta risk becomes.

The *power* of the experiment = $1 - \beta$

Interaction Effects

The effect of one independent variable on the dependent variable of interest may not be the same at all levels of the other independent variable, i.e., the effect of one independent variable may depend on the level of the other independent variable.

In order to find an interaction, you must have a factorial design, in which the two (or more) independent variables are "crossed" with one another so that there are observations at every combination of levels of the two independent variables.

Fractional factorial designs are experimental designs consisting of a carefully chosen subset (fraction) of the experimental runs of a full factorial design. The subset is chosen so as to exploit the sparsity-of-effects principle to expose information about the most important features of the problem studied, while using a fraction of the effort of a full factorial design in terms of experimental runs and resources.

Drawback: can only be used when the factors do not interact significantly in the ranges involved, otherwise, confounding occurs (i.e., when it is not clear if the effect is due to the factor, or the interaction between other factors)

Balanced Designs

An experimental design is balanced when each setting of each factor appears the same number of times with each setting of every other factor.

Resolution

An important property of a fractional design is its resolution or ability to separate main effects and low-order interactions from one another. Formally, the resolution of the design is the minimum word length in the defining relation excluding (1). The most important fractional designs are those of resolution III, IV, and V: Resolutions below III are not useful and resolutions above V are wasteful in that the expanded experimentation has no practical benefit in most cases—the bulk of the additional effort goes into the estimation of very high-order interactions which rarely occur in practice.

1. Resolution III designs have main effects confounded with 2-factor interactions
2. Resolution IV designs have main effects confounded with 3-factor interactions, and 2-factor interactions confounded with each other
3. Resolution V designs have some 2-factor interactions confounded with 3-factor interactions and some with 4

Remember: full factorial designs have no confounding.

CRE Part 5: Lifecycle Reliability

One-Factor Experiments

A	B	C	D	E	F	G
B	C	D	E	F	G	A
C	D	E	F	G	A	B
D	E	F	G	A	B	C
E	F	G	A	B	C	D
F	G	A	B	C	D	E
G	A	B	C	D	E	F

Test options for 1-factor experiments:

- Randomized testing
- Run in batches (e.g., temperature blocks)
- ***Latin-Square design***: allow for two blocking factors, i.e., these designs are used to simultaneously control (or eliminate) two sources of nuisance variability. These designs require that the number of levels of each of the noise factors is equal to the number of treatments. It is a table filled with n x n different symbols such that each symbol occurs exactly once in each row and each column.

Full-Factorial Experiments

A 2^2 full-factorial completely randomized experiment with results and mean results

Run #	A	B	A x B	Response, y			y(mean)
1	-	-	+	28.3	28.6	28.2	28.4
2	-	+	-	33.5	32.7	32.9	33.0
3	+	-	-	24.6	24.6	24.8	24.7
4	+	+	+	37.2	37.6	37	37.3

The main effect of factor A is (24.7+37.3)/2 - (28.4+33.0)/2

= 0.3

The main effect of factor B is (33+37.3)/2 - (28.4+24.7)/2

= 8.63

The interaction effect of A x B is (28.4+37.3)/2 - (33+24.7)/2

= 4.0

Is the interaction effect of A x B significant?

Perform ANOVA analysis to find out.

	-	+
A -	28.3	33.5
	28.6	32.7
	28.2	32.9
A +	24.6	37.2
	24.6	37.6
	24.8	37

The P-value column contains information relating to the statistical significance of the factors. P is the probability that the source of variation is not significant.

Robustness

Robustness means resistance to the effect of variation of some factor (e.g., resistance to change in temperature). Products and processes that are robust to noise of various kinds are clearly desirable. Genichi Taguchi developed techniques for improving robustness of products/processes:

Signal-to-noise Ratio

Taguchi experiments often use a 2-step optimization process. In step 1 use the signal-to-noise ratio to identify those control factors that reduce variability. In step 2, identify control factors that move the mean to target and have a small or no effect on the signal-to-noise ratio.

The SNR measures how the response varies relative to the nominal or target value under different noise conditions. Choose from different SNRs, depending on the goal of your experiment:

If it is desirable to maximize the mean,

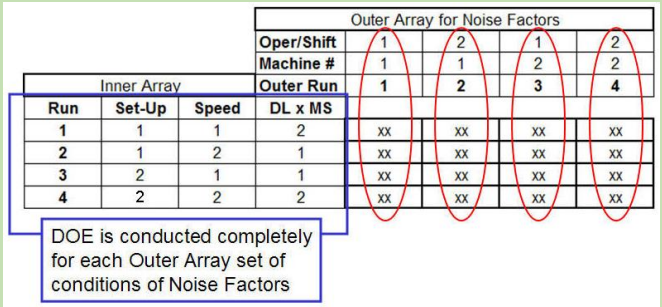
Signal-to-noise ratio (S/N) = mean / standard deviation

If it is desirable to minimize the mean,

Signal-to-noise ratio (S/N) = 1 / (mean x standard deviation)

Inner/Outer Array Design

This design distinguishes between 'noise' factors and 'control' factors. Control factors are put in the inner array and noise factors in the outer array. This design intentionally causes perturbations in the uncontrollable factors to find level combinations for the controllable factors that will minimize the variation in the desired output. This design determines optimum levels for the control factors in the presence of variation in the outer array factors.



4. Reliability Optimization

The standard ways to improve reliability, redundancy, derating and environmental robustness tend to increase cost and weight, and delay the design process. A ***prioritization matrix*** can be used to optimize the trade-offs.

Reliability optimization process: a reliability tool that analyzes a system to help determine which components will meet the performance goals at the lowest cost.

3 steps:

1. Develop a model that represents the entire system, by constructing a system RBD that represents the reliability relationships of the components in the system
2. Using this model estimate the reliability and costs that would be incurred in the process of making modifications
3. Then perform an optimization analysis to find the best combination of component reliability improvements that meet or exceed the performance goals at the lowest cost.

CRE Part 5: Lifecycle Reliability

5. Human Factors

Reliability of the human aspect can be improved by:

- Following human-factor principles in system design
- Careful selection and training of personnel

Stress affects human performance and reliability, but is not necessarily an entirely negative state. Stress at a moderate level is useful in increasing human effectiveness to its optimal level.

4 types of occupational stressors are:

1. Work overload/underload
2. Occupational change
3. Occupational frustration
4. Other sources (e.g., noise, poor relationships)

Human Performance Reliability Modeling

$$R_e(t) = exp\left[-\int_0^t h_e(t)dt\right]$$

Where $R_e(t)$ is the human performance reliability at time t and $h_e(t)$ is the time-dependent human error rate.

Human Reliability Data Sources

1. Past experience
2. Customer's failure reporting system
3. Product development data
4. Repair reports
5. Field installation testing
6. Manufacturing and quality control
7. Acceptance tests in the factory
8. Field demonstration and environmental qualification tests

Human Factors must be considered from the preliminary design phase, to the advanced design phase, mock-up of prototype fabrication phase and test and evaluation phase. Reliability testing must consider variation introduced by differences between people, such as:

- Usage factors – people use items differently
- Installation factors – lifetime can be affected by quality of installation and location environment
- Process management factors – variations can be introduced by people in the production process

The product design team should strive for robustness to the variation introduced by these human factors, keeping the specs as broad as possible. Reliability testing procedures must include tests throughout the specification spectrum.

6. Design for X (DFX)

Typical design constraints:

Design for Assembly: conscious thought given to the assembly process by the product design team.

Design for Manufacturability: puts more emphasis on the fabrication or primary functions that precede assembly.

Design for Testability: ensures design characteristics under test are easily and accurately measured.

Design for Cost: strong constraint put on final product cost.

Design for Serviceability: considers ease/simplicity of installation and servicing.

Design for Reliability: emphasizes the long-term usefulness of the product to the customer.

Design for Environment: considers recycling, reclamation, disposal.

The testing requirements for a system should provide for a method to detect a system fault and isolate the failed component, e.g., built-in tests, easily accessible test points. When designing for testability, it is important to consider the various conditions when testing is required (e.g., during design, maintenance, failure analysis).

Fault detection capability: a measure of the faults detected by the fault detection system (e.g., built-in test) compared to the total number of system faults.

Fault isolation capability: a measure of the % of time the failure can be isolated to a given number of replaceable components. Can be accomplished by a diagnostic analysis, built-in test, or using external equipment.

False-alarm rate: a measure of the rate at which the system declares the detection of a failure when no failure occurred.

A *fault recording and analysis system* that identified specific components will aid in improving reliability.

7. Design for Reliability (DfR)

Reliability design is an iterative process that begins with the specification of reliability goals consistent with customer requirements.

Design for Reliability (DfR) is a concurrent engineering program where the reliability engineer is part of the product development team working with the design engineers to design reliable products with overall low lifecycle costs. A DfR program encompasses the following:

- Reliability program planning
- Reliability predictions
- Parts derating
- Thermal analysis
- Component wear-out
- FMEA and fault tree analysis
- Availability and system modeling
- HALT, HASS, and HASA
- Design verification testing (DVT)
- Product return rate analysis
- FRACAS and root cause failure analysis
- Internal process sharing and benchmarking

Reliability Specification and System Measurements are usually designated by MTTF or MTBF. This parameter is not sufficient except in case of exponential failure distribution. A better measure would be to specify the reliability at specific points in time. If the MTTF is to be the only reliability spec used, a constant failure rate should be assumed and subsequently demonstrated.

Considerations in Designing in Reliability

Fault Tolerance: the ability of a system to perform according to specifications even when undesired changes occur in its internal or external environment.

Various techniques produce fault-tolerant systems:

Redundancy: if 2 or more parallel channels are available for the system, the likelihood of failure is reduced. Redundancy alone if often insufficient because the common cause variation may overwhelm all branches of the system.

Diversity and *separation* can be used to make redundancy more effective:

Diversity: used to construct parallel channels different in ways that will make them immune to failure by the same cause. This, however, implies an understanding of the common causes.

Separation: can apply to branches that have separate physical environments, electronic isolation, or other means of preventing a common cause from causing a multiple-branch failure.

Reducing variation in design is essential to success and applies not only to physical tolerances, but all portions of the product and process. Taguchi design of experiments methods and Monte Carlo simulation are powerful tools used in parameter design to reduce design variation.

Part 5B: Parts and Systems Development

1. Materials and Components Selection Techniques

In some fields (e.g. electronics), standard ratings of stress limits (temp, voltage, pressure, etc.), environmental conditions and other characteristics are available for components. Operating beyond the rated value risks part damage or failure.

Derating is the practice of operating components at lower stress levels than those specified by the standards. It usually refers to limiting electrical, thermal or mechanical stresses.

Underclocking of synchronous electronic devices is a type of derating (e.g., running a CPU at a slower speed to reduce heat generation and prolong life).

Stress-Life Relationships

The analysis of reliability data can be used to generate a life distribution, typically defined for a given stress level. There may be a different life distribution for different stress levels.

2. Parts Standardization and System Simplification

Parts standardization: the use of the same components in several products.

A *parts reduction* initiative makes an effort to reduce the number of parts required to perform a function, and hence simplify parts, assembly, and maintenance.

Parallel models are product families which are essentially the same except for size.

Raw as possible (RAP) principle: when, during the fabrication process, parts are maintained in a state where they may be used in several models and then customized as needed.

All of these techniques provide **cost advantages** in terms of inventory, purchasing efficiencies, storage, accounting, auditing as well as reduced reliability testing.

Software reuse: software should only be reused if the new use is within its tested capability.

Part 5C: Maintainability

1. Maintenance Strategies

Maintenance strategies should be chosen to ensure a high level of availability while controlling costs.

The information needed to choose the most appropriate replacement strategy includes:

- The failure distribution of the unit
- The cost associated with unit failure
- Any safety issues associated with the failure
- The cost of replacing the unit
- The cost of scheduled replacements
- The cost of inspection or test

Predictive & Reliability-Centered Maintenance

Predictive Maintenance assumes the operator can detect the imminent failure of a unit, by observation, analysis, or using test equipment.

Reliability-Centered Maintenance uses the predicted failure distributions to determine the optimum replacement time for units about to enter the wear-out phase.

Note that maintenance actions can negatively impact reliability by introducing failure modes during the maintenance.

Maintainability/Apportionment/Allocation

Maintainability is the ability of an item to be retained in or restored to specified conditions when maintenance action is performed by personnel having specified skill levels and using prescribed procedures and resources at each prescribed level of maintenance and repair.

Maintainability ***allocation*** is a continuing process of apportioning requirements at the system level to subsystem levels, during design. To do this, the failure rate and MTTR are used:

e.g., for a series system with 3 subsystems,

Failure rate for subsystem *i* is λ_1, and the MTTR is t_i

System failure rate $\lambda = \lambda_1 + \lambda_2 + \lambda_3$

$$MTTR_{System} = t = \frac{(\lambda_1 t_1 + \lambda_2 t_2 + \lambda_3 t_3)}{(\lambda_1 + \lambda_2 + \lambda_3)} = \frac{\lambda_1 t_1}{\lambda} + \frac{\lambda_2 t_2}{\lambda} + \frac{\lambda_3 t_3}{\lambda}$$

The system-level MTTR requirement is t^*. In order to meet the maintainability goal, an MTTR allocation to each subsystem must be made if the $MTTR_{System} > t^*$. The MTTR allocation to subsystem *i* is,

$$t_i^* = [t_i / t] \times t^*$$

Spare Parts Analysis

The number of spares required is dependent on the number of failures and the amount needed for preventive maintenance.

The MTTR calculation assumes spare parts are readily available. If the unit has a constant failure rate, the probability of requiring no more than *r* replacement units can be found using the cumulative Poisson distribution.

$$P(r) = \sum_{n=0}^{r} \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Several methods are in use for forecasting spare parts requirements:

Moving Average

The forecasted value of the spare parts requirement is the average of the previous *n* requirements. The value of *n* is determined by experience.

$$R_t = \frac{X_{t-1} + X_{t-2} + \cdots + X_{t-n}}{n}$$

Where,

R_t = forecasted spare parts requirement at time *t*

X_t = actual spare parts requirement at time *t*

Weighted Moving Average

This method weights the most recent values higher than the previous values. The *n*-period weighted moving average is given by,

$$R_t = \frac{nX_{t-1} + (n-1)X_{t-2} + \cdots + X_{t-n}}{n + n - 1 + \cdots + 1}$$

Single Exponential Smoothing

$$R_t = \alpha X_{t-1} + (1-\alpha) R_{t-1}$$

Where,

α = smoothing factor, usually between 0.1 and 0.4 with unstable demand situations using a higher α-value. Experience dictates the best values for α.

Croston Method

The Croston method applies exponential smoothing to both the magnitude of the spare parts requirement (*Z*) and the time period between demands for the spare part (*P*). If no spare part is needed at the end of the review period, the values of *Z* and *P* are unchanged. If, however, X_t spare parts are needed at time *t*, the revised estimates for *Z* and *P* are given by the smoothing formulas,

$$Z_{t+1} = \alpha X_{t+1} + (1-\alpha) Z_t$$

$$P_{t+1} = \beta G_{t+1} + (1-\beta) P_t$$

Where,

X_t = actual value of demand at time *t*

Z_t = forecasted value of demand at time *t*

G_t = actual demand between previous demand and this demand

P_t = forecasted value of time between previous demand and this demand

α, β = smoothing factors between 0 and 1

Then, the forecast value for number of parts required per period for the next period would be,

$$R_{t+2} = \frac{Z_{t+1}}{P_{t+1}}$$

Planning

Repairable systems: systems that can be restored to service via corrective maintenance actions following a failure.

Preventive Maintenance (PM): includes all the actions performed to keep the system in an operating state by preventing wear-out failures. Does not reduce inherent failure rate, but maintains the system at that level of failure probability. Can be planned.

Corrective Maintenance (CM): includes all the actions required to return the system to an operating state once failure has occurred. Cannot be planned; may be deferred.
Availability: a measure of the likelihood that a system will be ready to operate when it is called on to operate.

Availability = uptime / total time

Availability is a function of reliability and maintainability:

- *Reliability* deals with reducing the frequency of unscheduled downtime
- *Maintainability* deals with reducing the duration of downtime (for scheduled and unscheduled actions)

Mean time to repair (MTTR): the mean time required to bring a system back to an operable state after a failure:

$$MTTR = \frac{\Sigma(\lambda_i t_i)}{\Sigma\lambda_i}$$

Where,
λ_i = failure rate for the *i*th failure mode,
t_i = time to repair the system after the failure has occurred
The **lognormal** distribution is usually used to model times to repair. Most repairs can be made in a time less than the mean.

$$MTTR = e^{\left(\mu + \frac{\sigma^2}{2}\right)}$$

In order to increase the availability of a system, the system reliability could be increased (increase MTBF) or the time to restore the system to service could be reduced (decrease MTTR).

2. Preventive Maintenance (PM) Analysis

The goal of PM is to optimize system reliability. Cost savings can be made if PM (replacement) is performed on systems with increasing (ageing) failure rates. An optimum maintenance interval can be determined by considering:

1. The time-to-failure distribution and the parameters of that distribution
2. The effect on the system of the failure
3. The cost of the failure (incl. cost of downtime)
4. The cost of scheduled maintenance, including the cost of the replaced unit
5. The effect scheduled maintenance has on reliability – will the activity introduce failures into the system?
6. Is the potential failure detectable by an operator? Can the operator take corrective action before the failure propagates throughout the system causing other failures?
7. The cost of inspection and testing

3. Corrective Maintenance Analysis

Corrective maintenance is performed in the event of a failure or malfunction of the system. It cannot be planned. The total downtime includes active maintenance time and inactive/delay time. The **active corrective maintenance** time can be analyzed as 7 steps:

1. *Localization*: determining the system fault without using test equipment
2. *Isolation*: verification of the system fault using test equipment (or built-in tests)
3. *Disassembly*: accessing the fault
4. *Interchange*: replacing or repairing the fault
5. *Reassembly*: rebuilding the item
6. *Alignment*: adjusting the item per specifications
7. *Checkout*: performing standard QA checks to verify correct operation

Good design practice would dictate that repairs with long MTTR values will rarely be necessary.

References

This study guide is a summary of ***The Certified Reliability Engineer Handbook, Third Edition***
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Page	Image Reference Links
2	House of quality: http://asq.org/img/laq/house-of-quality-figure-lg.gif
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4	Bathtub curve: http://www.sdram-technology.info/pictures/semiconductor-reliability-bathtub-curve.jpg Software reliability curve: https://users.ece.cmu.edu/~koopman/des_s99/sw_reliability/Image2.gif
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7	FMECA: http://www.cqeacademy.com/wp-content/uploads/2016/05/FMECA-Critical-Analysis-Matrix-with-Failure-Modes.png
8	Hierarchy of controls: https://www.cdc.gov/niosh/topics/noise/images/HOC.png
9	Control Chart example: http://asq.org/learn-about-quality/data-collection-analysis-tools/overview/control-chart.html
10	Contingency table: http://study.com/cimages/multimages/16/con1.jpg
12	Weibull graph: http://www.weibull.com/hotwire/issue14/ch6_fig5.gif
13	Control chart: https://www.spcforexcel.com/files/images/ptbeyond_0.gif
20	Duane Model: http://reliawiki.org/index.php/Duane_Model
22	OC Curve: http://support.minitab.com/en-us/minitab/17/topic-library/quality-tools/acceptance-sampling/acceptance-sampling-graphs/operating-characteristic-oc-curve/ Sequential test: https://src.alionscience.com/pdf/SequentialReliabilityTests.pdf
23	RBDs: http://www.mdpi.com/energies/energies-07-00238/article_deploy/html/images/energies-07-00238f3-1024.png
24	Standby model: http://www.weibull.com/hotwire/issue22/rb22_1.gif
28	Product Life Cycle: http://cdn3.businesssetfree.com/wp-content/uploads/2013/04/Product-Lifecycle-e1364967651462.png?x89729 Stress-Strength: CRE Prep Course Presentation notes by James Breneman
30	Inner/Outer Array Design: https://www.educatevirtually.com/Media/Default/Blog/DOE_Inner_Outer_Arrays.jpg