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ANALYSIS OF SURVIVAL DATA IN ENGINEERING, BUSINESS, AND MEDICINE

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Purpose. To show applications of survival data analysis in Engineering, Business, and Medicine. To present

- Basic concepts and applications of survival/life data analysis,
- The Weibull distribution and its failure rate function,
- Probability plots to get useful information from survival data,
- Various applications.

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OVERVIEW

APPLICATIONS in engineering, medicine, and business

WEIBULL DISTRIBUTION for survival data

STENT BALLOONS – meet reliability specifications?

IRONING CORDS – which is better?

GENERATOR WINDINGS – is preventive replacement needed?

LOCOMOTIVE DISKS – least cost of replacement policy?

TOASTERS – how much will eliminating failure mode 11 improve toaster life?

CONCLUDING REMARKS

REFERENCES

MEDICAL APPLICATIONS

- Clinical studies where patient survival is an important measure of treatment effectiveness.
- Life of medical devices and components, e.g.,
  - When should an aging heart pacemaker be replaced to prevent failure in a patient?
  - At what rate should x-ray tubes be manufactured to replace those that wear out in service?
  - How many times during surgery can an angioplasty balloon be re-inflated without bursting?

BUSINESS APPLICATIONS

- Nielson TV rating service employs a sample of households. There is a distribution of the time when households leave the sample. At what rate must new households be acquired?

- Banking Corp.A bought Banking Corp.B. Because accounts in Corp. B close over time, the IRS treats those accounts as a depleting asset for tax purposes of Corp. A. The distribution of time to closure was needed.

- TeleCommunications Inc. acquired four cable TV systems and their subscribers. The distribution of time when subscribers terminate service determines the depreciation that the IRS will allow.

ENGINEERING APPLICATIONS

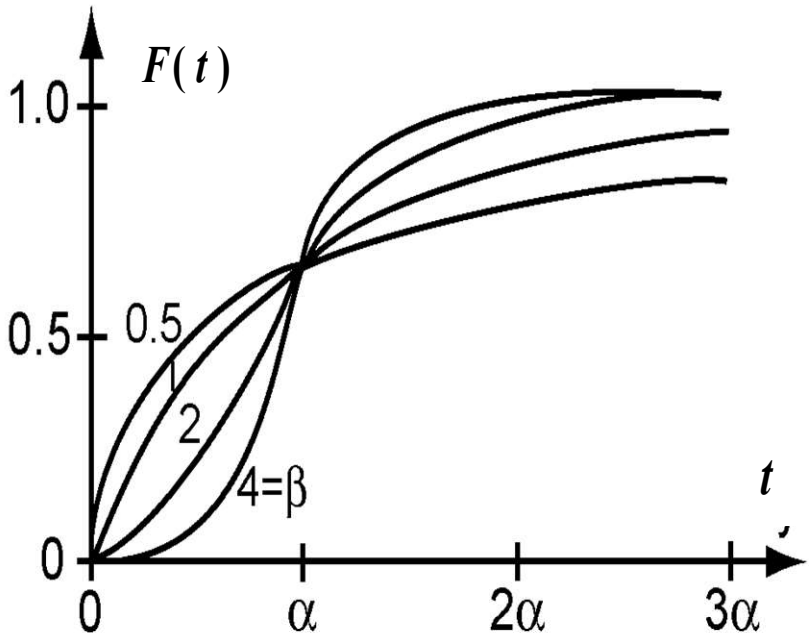
Engineers and managers need to know the life distribution of consumer, industrial, and military products for various reasons:

- To determine if the life distribution of a new product under development satisfies customer requirements.
- To plan for the cost of failures during warranty.
- To determine if a product's life satisfies the customer.
- To decide when preventive replacement should be used.
- To compare product life with different designs, materials, manufacturing techniques, operating conditions, etc.

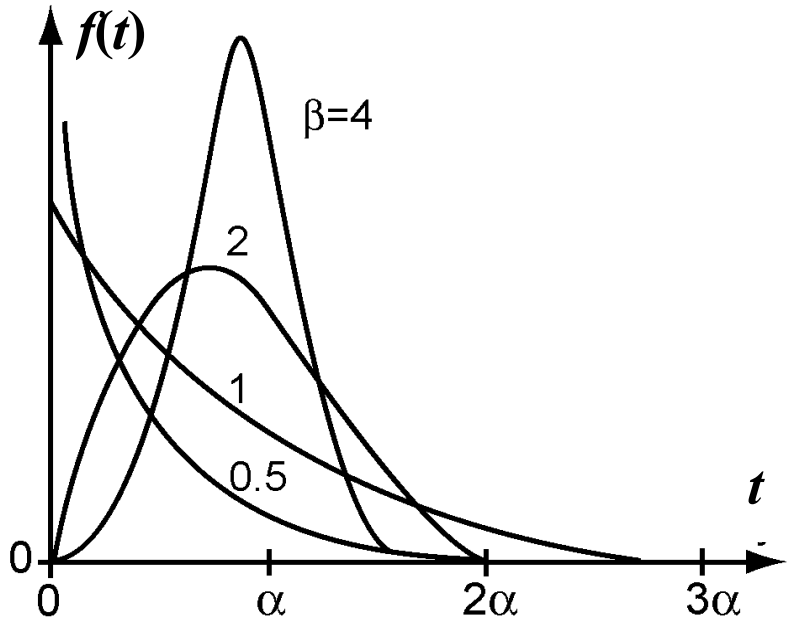
WEIBULL DISTRIBUTION often is a good model for life data

The Weibull cdf for the population fraction failed by age t is

$$F(t) = 1 - \exp[-(t/\alpha)^{\beta}], \qquad t > 0 .$$

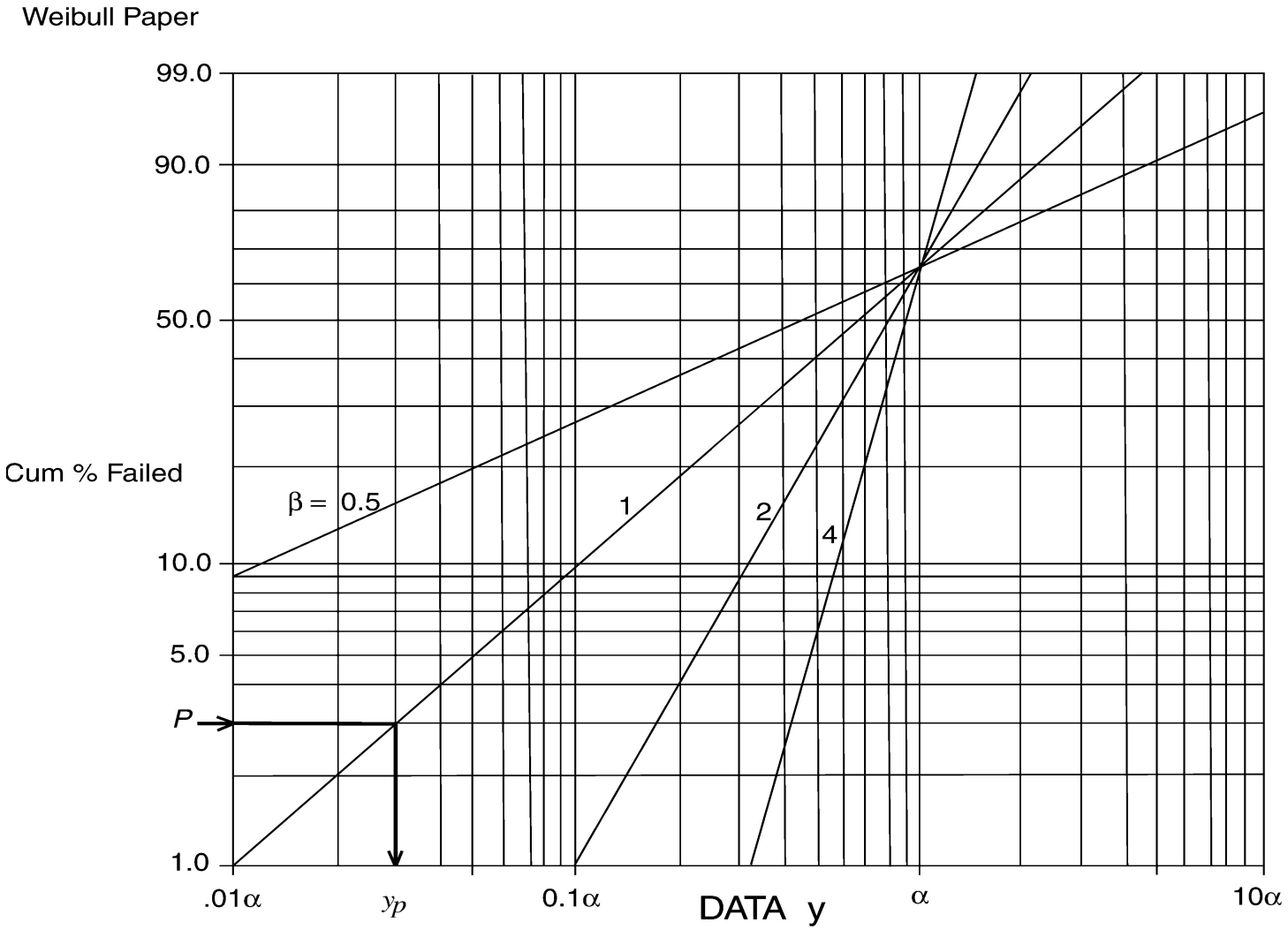


Weibull cdfs.



Weibull probability densities.

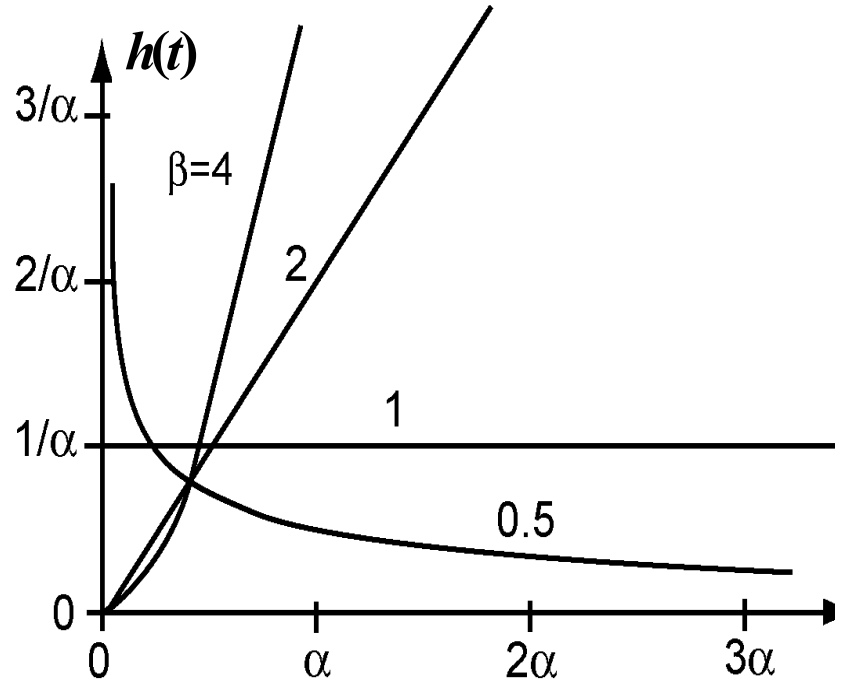
Weibull Plot of cdf's



The Weibull population ***hazard function*** at age t is

$$h(t) = (\beta/\alpha^{\beta})\, t^{\beta-1}\ ;$$

it is also called the instantaneous ***failure rate*** *function*.



Weibull hazard functions.

Complete data

30 STENT BALLOONS

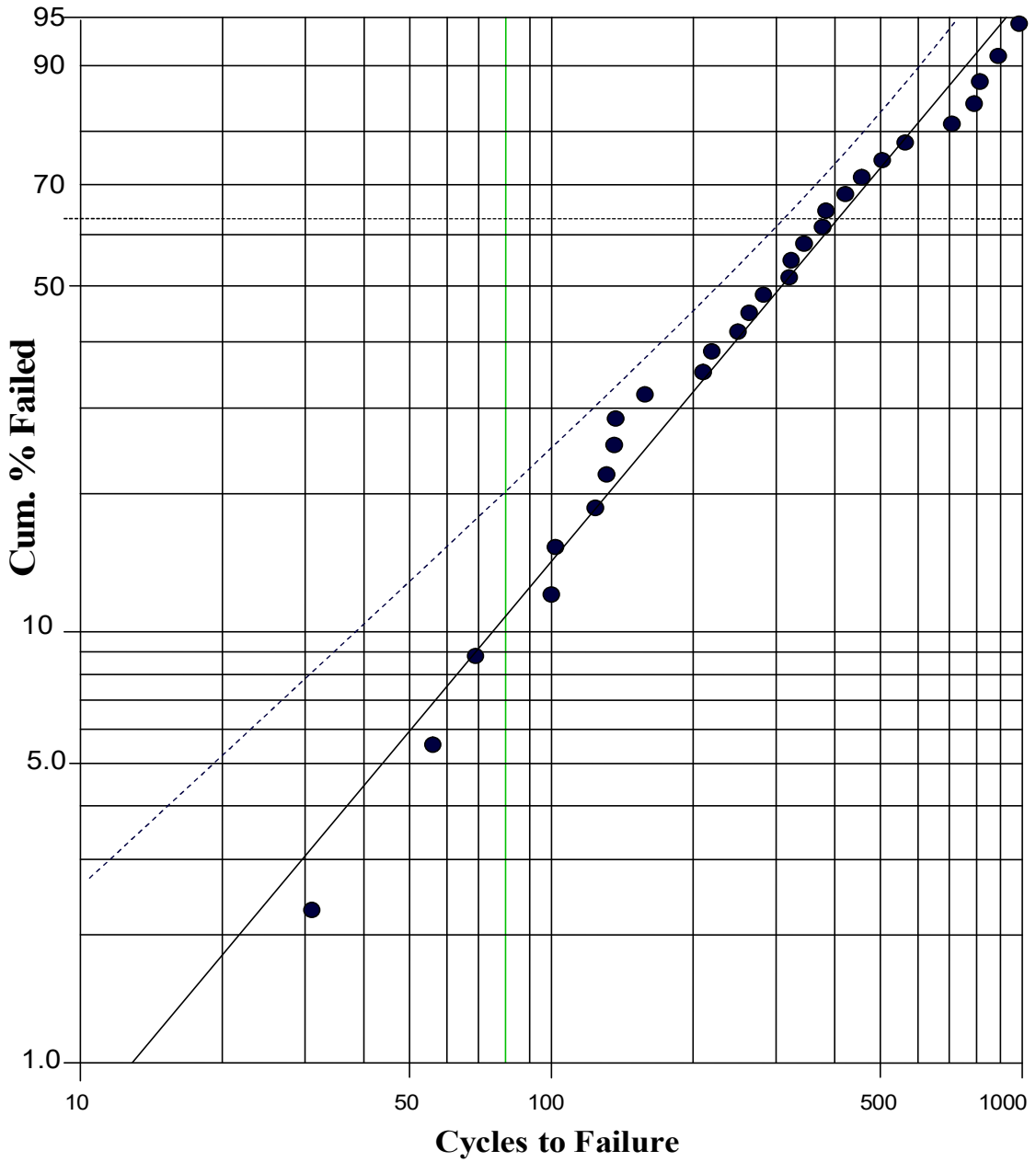
-- Cycles to Failure

Cum.% = 100 i / (n+1)

31	131	249	377	709
56	136	263	383	790
69	137	282	421	813
100	158	320	456	889
102	210	323	504	984
124	219	344	564	1028

Demonstrate:

- 90% reliability
- at 40 cycles with
- C^*=95% confidence

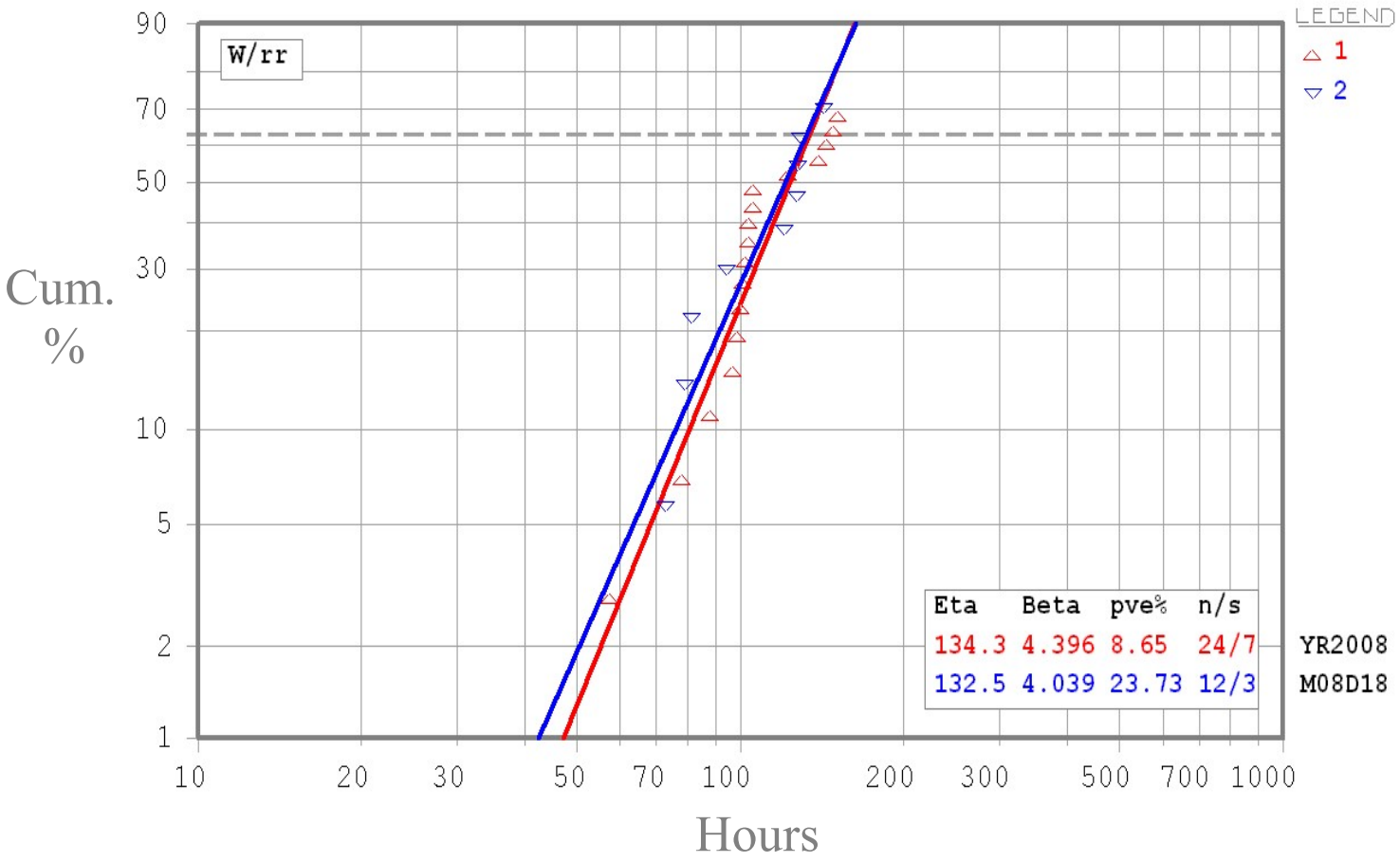


COMPARISON OF TWO IRONING CORDS ON FLEX TEST

Type B6 (New)				Type B7 (Old)		
Hours	Rank	$100i/(n+1)$		Hours	Rank	$100i/(n+1)$
57.5	1	4.0		72.4	1	7.7
77.8	2	8.0		78.6	2	15.4
88.0	3	12.0		81.2	3	23.1
96.9	4	16.0		94.0	4	30.8
98.4	5	20.0		120.1	5	38.5
100.3	6	24.0		126.3	6	46.2
100.8	7	28.0		127.2	7	53.8
102.1	8	32.0		128.7	8	61.5
103.3	9	36.0		141.9	9	69.2
103.4	10	40.0		164.1+	10	–
105.3	11	44.0		164.1+	11	–
105.4	12	48.0		164.1+	12	–
122.6	13	52.0				
139.3	14	56.0				
143.9	15	60.0				
148.0	16	64.0				
151.3	17	68.0				
161.1+	18	–				
161.2+	19	–				
161.2+	20	–				
162.4+	21	–				
162.7+	22	–				
163.1+	23	–				
176.8+	24	–				

These data are singly censored

WEIBULL PLOT OF LIFE OF OLD AND NEW CORDS



The two cords have the same life distribution. The New cord was cheaper and was adopted to reduce cost.

LIFE (IN MONTHS) OF FIELD WINDINGS OF 16 LARGE GENERATORS (+ is unfailed)

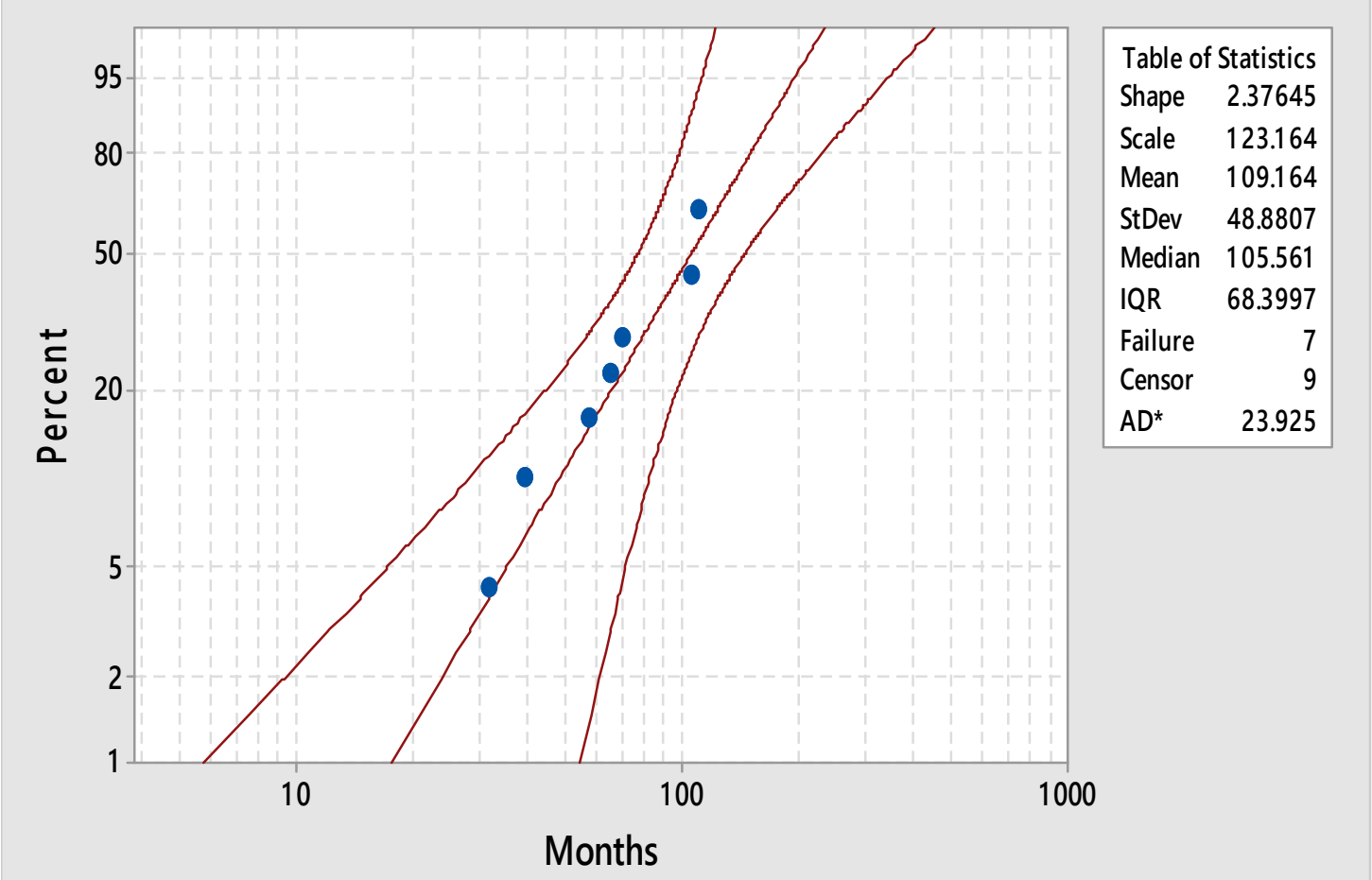
31.7	39.2	57.5	65.0+	65.8	70.0	75.0+	75.0+
87.5+	88.3+	94.2+	101.7+	105.8	109.2+	110.0	130.0+

Do they have an increasing or decreasing failure rate?

If increasing, preventative replacement is needed to avoid in-service failure.

These data are multiply censored. See the references for the plotting positions, which are complicated

Weibull Plot of Generator Winding Life



$\beta^* = 2.4 > 1$. Thus the failure rate is increasing and preventive replacement is needed. When?

DATA ON LOCOMOTIVE DISK LIFE IN SERVICE

kmiles	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Failed				**1**	**1**	**3**			**1**		**1**		**2**	**1**			**1**	**1**				**1**						
Running	**1**	**1**	**2**	**2**	**11**	**10**	**17**	**15**	**10**	**11**	**9**	**9**	**9**	**5**	**13**	**14**	**14**	**14**	**10**	**5**	**3**	**2**		**4**		**1**		**1**

$\beta^* \approx 2$. Disks must be replaced. Which decision costs least?

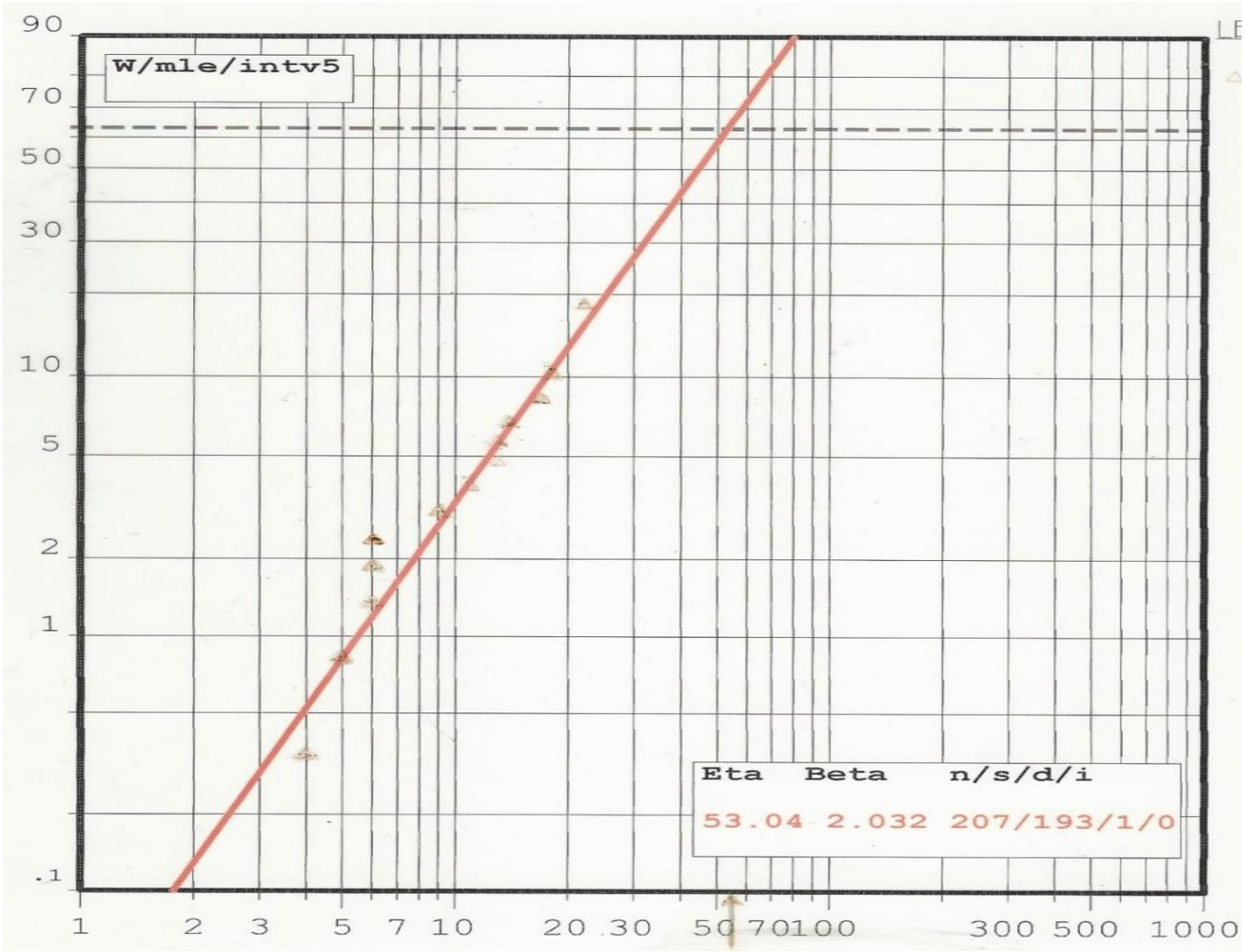
1. No preventive: Replace a disk only when it fails.
2. At scheduled overhaul 55k miles, replace all disks.
3. Replace all disks immediately.

Replacing a disk with a new design that does not fail:

a) before disk failure, costs \$3000,

b) after disk failure, costs \$6000 due to damage to engine.

WEIBULL PLOT OF DISK LIFE



1000 Miles

COST OF THE DECISIONS

$\beta^* \approx 2.0 > 1$ indicates the failure rate increases. High mileage disks should be replaced first to reduce failures

1. **Not replace:** $6,000 x 193 = $ 1.2 M.

2. **At 55k mi.:** $6,000x0.64x193 + $3,000x0.36x193 = $ 0.9 M.

3. **Replace now:** $3,000 x 193 = $ 0.6 M. Least cost.

TOASTER LIFE DATA

How much does eliminating mode 11 improve toaster life?

0 = censored life

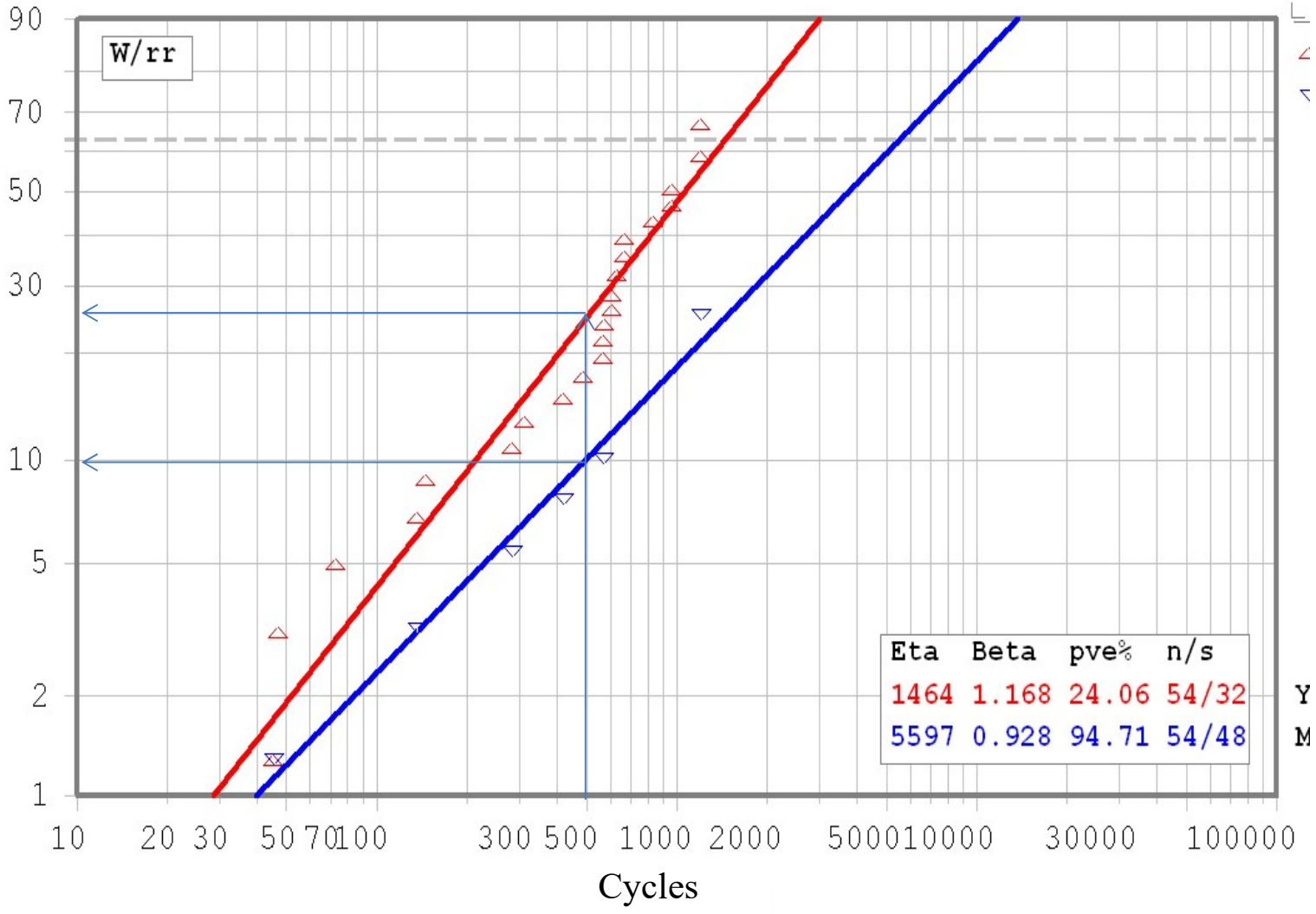
Treat mode 11 as a censoring time when mode 11 is eliminated

Multiply censored data with competing failure modes.

Cycles	Mode		Cycles	Mode
45	1		608	0
47	11		608	0
73	11		608	0
136	6		608	0
136	0		608	0
136	0		608	0
136	0		608	0
136	0		608	0
145	11		630	11
190	0		670	11
190	0		670	11
281	12		731	0
311	11		838	11
417	12		964	11
485	0		964	11
485	0		1164	0
490	11		1164	0
569	1		1164	0
571	11		1164	0
571	0		1164	0
575	11		1164	0
608	11		1164	0
608	11		1198	11
608	0		1198	9
608	0		1300	0
608	0		1300	0
608	0			

Eliminating Mode 11 reduces the % failing on warranty (500 cycles) from 27% to 11%.

With mode 11 Without Mode 11



OTHER INFORMATION FROM LIFE DATA ANALYSIS

- Predictions and prediction limits for future numbers of failures of units in service. This information is used to plan manufacture and stocking of replacement units and parts.

- Analysis of data with competing failure modes, often with a curved Weibull plot. Each failure mode has its own life distribution.

- Analysis of multiple inspection data where units are periodically inspected for failure. One knows only that a failure occurred between the inspection time the failure was found and the previous inspection time.

- Analysis of single inspection data where units are inspected only once. Then one knows only that a failure occurred before or after the inspection.

- Estimation of system reliability using component life data.

- Nelson (2004) reviews the features of commercial software that analyze life data – distribution fitting and data plots.

See the references for such information.

CONCLUDING REMARKS

Such numerical and graphical analyses provide information needed to make good decisions from life data.

REFERENCES

Meeker, William Q., Escobar, Luis A., and Francis G. Pascual (2021), ***Statistical Methods for Reliability Data***, 2nd ed., Wiley, New York.

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Dr. Nelson is a leading expert on analysis of reliability and accelerated test data. For 24 years he consulted across the General Electric Co. and received the Dushman Award of GE Corp. R&D for developments and applications of product reliability data analysis. He now privately consults on applications, gives training courses, and works as an expert witness.

He is a Fellow of the Amer. Statistical Assoc. (1973), the Amer. Soc. for Quality (1983), the IEEE (1988), elected for his innovative statistical developments for reliability data analysis and for contributions to reliability education. He was awarded the 2003 Shewhart and 2010 Shainin Medals of ASQ, and the 2005 Lifetime Achievement Award of the IEEE Reliability Society for his developments of reliability methodology and contributions to reliability education. He received the 2018 Hahn Award of ASA for outstanding service to clients. He has received nine Outstanding Presentation Awards from ASA.

He authored three highly regarded books *Applied Life Data Analysis* (Wiley 1983), *Accelerated Testing* (Wiley 2004), *Recurrent Events Data Analysis* (SIAM-ASA 2003), two ASQ booklets, and 140 literature articles. He can be reached at WNconsult@aol.com.